



Math Olympiad and Problem Solving Programs
E230 - Advanced Math Competitions
Problem Set 15.1 - Arithmetic Sequence and Series

Name:

Date:

1. (a) $\boxed{43}$

(b) $\boxed{1 + 3(n - 1) = 3n - 2}$

2. $\boxed{15}$

3. $\boxed{95}$

4. We can notice that the series is the sum of multiples of 7 so we can factor out 7:

$$\begin{aligned} 21 + 28 + 35 + \cdots + 105 &= 7(3 + 4 + 5 + \cdots + 15) \\ &= 7 \left[\frac{15(16)}{2} - 1 - 2 \right] \\ &= \boxed{819} \end{aligned}$$

5. $\boxed{-175}$

6. $\boxed{-3}$

7. Let's represent the arithmetic series by $a_n = a_1 + k(n - 1)$. Then the sum of the first 5 terms equalling 70 gives the following equation:

$$\begin{aligned} a_1 + a_2 + a_3 + a_4 + a_5 &= 5a_1 + k(0 + 1 + \cdots + 4) \\ &= 5a_1 + 10k = 70 \end{aligned}$$

The sum of the first 10 terms equalling 210 gives the following equation:

$$\begin{aligned} a_1 + a_2 + \cdots + a_{10} &= 10a_1 + k(0 + 1 + \cdots + 9) \\ &= 10a_1 + 45k = 210 \end{aligned}$$

We can solve this system of equations to get $k = \frac{14}{5}$ and $a_1 = \boxed{\frac{42}{5} = 8.4}$

8. $\boxed{8}$

9. $\boxed{300}$



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10. Notice first that if our sequence was represented by $a_n = a_1 + k(n-1)$ then $a_2 - a_1 = k$. We can replace numbers in our sum with a_1 and k . The first sum is easier, giving us $100a_1 + 99k = 200$. The second sum is a little trickier. Notice that $a_{101} = a_1 + 100k$:

$$100a_{101} + 99k = 200$$

$$100(a_1 + 100k) + 99k = 200$$

$$100a_1 + 10099k = 200$$

Now if we solve the system of equations to get $a_2 - a_1 = k = \boxed{\frac{1}{100} = 0.01}$.