Avid Academy for Gifted Youth

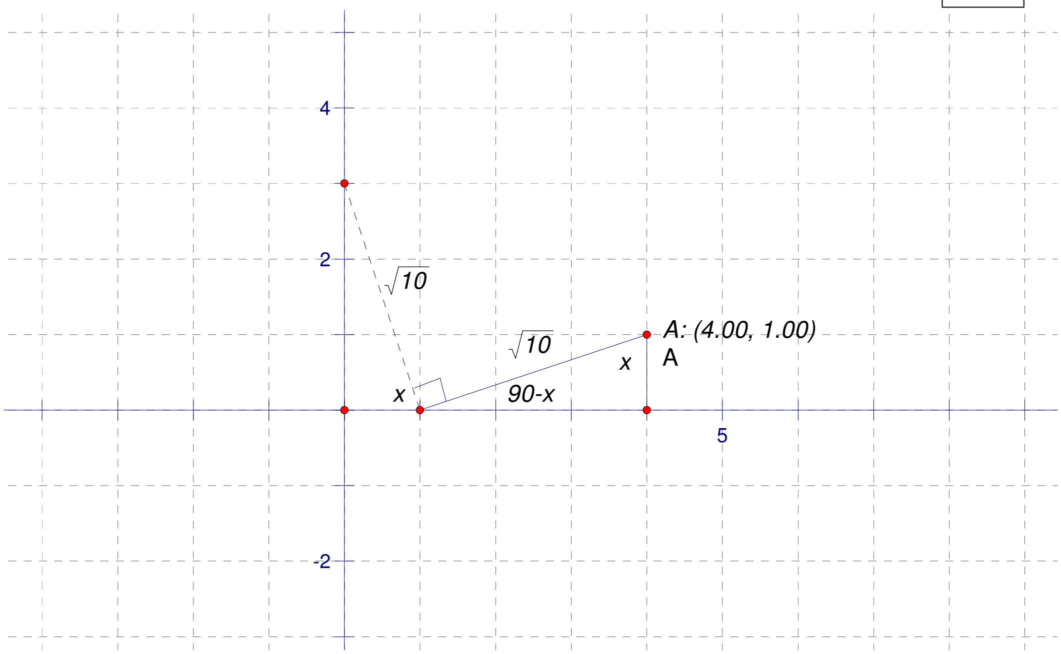
Math Olympiad and Problem Solving Programs
E230 - Advanced Math Competitions
Problem Set 14.2 - Coordinate Geometry
Name: Date:

1. $\boxed{(3.4, 4.8)}$

2. $\boxed{17}$

3. $\boxed{3:5}$

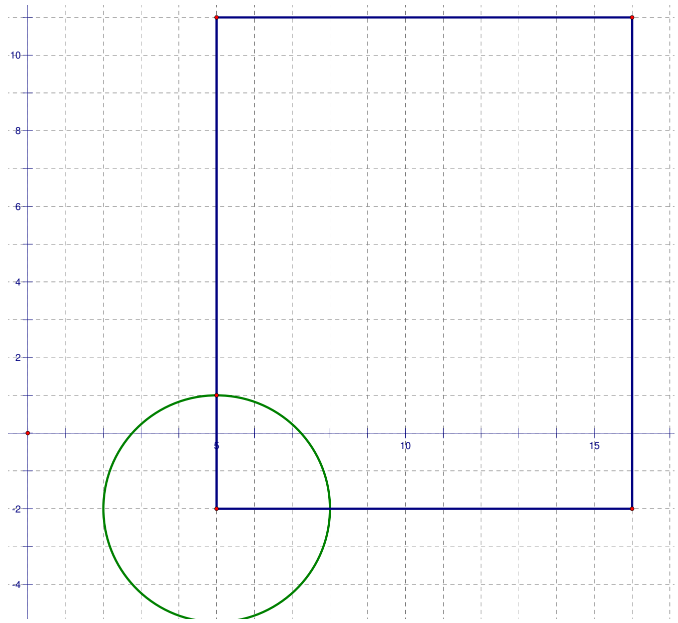
4. Draw the points $(4, 1)$ and $(1, 0)$ and connect them (which a line of length $\sqrt{10}$). Now we need to rotate our line counterclockwise 90 degrees. Notice that the angle we create by dropping a perpendicular from $(4, 1)$ to the x axis is angle x, and the other angle is $90 - x$ (see diagram). Then when we rotate, we see angle x again, so we have congruent triangles. Thus we know the height of our second triangle is 3, so the coordinate of the new point is $\boxed{(0, 3)}$.



5. $\boxed{(\frac{-15}{4}, \frac{41}{4})}$

6. $\boxed{(1, 7)}$

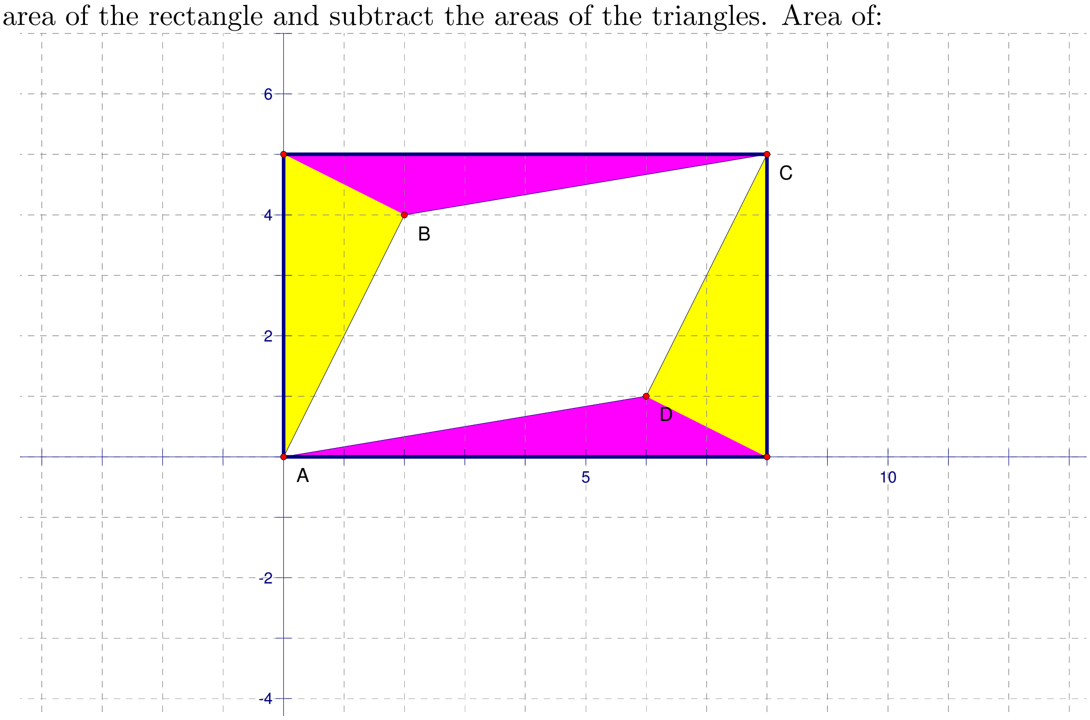
7. Our rectangle has vertices $(5, 11), (16, 11), (5, -2)$ and $(16, -2)$. The equation $(x - 5)^2 + (y + 2)^2 = 9$ describes a circle (to refresh your memory, and equation of the form $(x-k)^2+(y-h)^2 = r^2$ is a circle with center (k, h) and radius r). So our equation tells us that we have a circle with center $(5, -2)$ and radius 3. Draw this on the graph.



Obviously, the area we want is a quarter of a circle with radius 3. The area of the whole circle is 9π, so a quarter of it is $\boxed{\frac{9\pi}{4}}$

8. $\boxed{3}$

9. Draw the quadrilateral, then draw a rectangle surrounding the quadrilateral, and find the area of the rectangle and subtract the areas of the triangles. Area of:



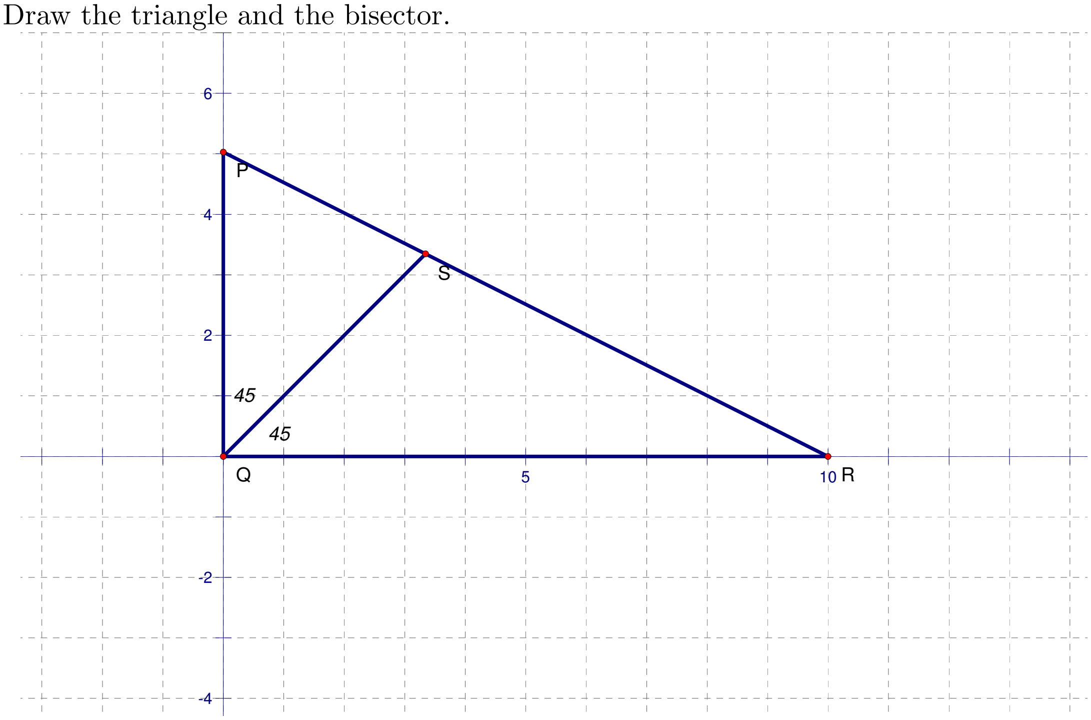
Rectangle: $5 \times 8 = 40$

Yellow triangles: $\frac{5\times 2}{2} = 5$

Pink triangles: $\frac{8\times 1}{2} = 4$

Thus, area of the quadrilateral is $40 - 2 \times 5 - 2 \times 4 = \boxed{22}$

10. Draw the triangle and the bisector.



We can find the length of PR by the Pythagorean Theorem, which gives us $PR = 55$. Also, from the angle bisector theorem, we know $\frac{|QP|}{|QR|} = \frac{|PS|}{|SR|} = \frac{5}{10} = \frac{1}{2}$ (the side bars mean ”‘length of”’). So basically, the lengths of segments PS and SR are in the ratio 1 : 2. So we know $|PS| = \frac{5}{3}\sqrt{5}$ and $|SR| = \frac{10}{3}\sqrt{5}$. However, that is not what we are looking for (but remember this ratio for the AMC10). Now the angle bisector theorem also gives us the following formula: $|QS|^2 = |QR||QS|(1 - \dfrac{|PR|^2}{(|QR| + |QS|)^2})$. Plugging in what we know, we get $|QS|^2 = 5 \times 10(1 - \frac{125}{225}) = 50 \times \frac{4}{9} = \frac{200}{9}$. So $|QS| = \boxed{\frac{10\sqrt{2}}{3}}$