

Name:

Date:

1.  $\boxed{-6}$

2.  $\boxed{1}$

3.  $\boxed{5}$

4. You should be familiar with polynomial division.

<p><b>The Set-Up</b></p> $5x - 2 \overline{) 5x^2 - kx + 7}$	<p><b>Step 1</b></p> $5x - 2 \overline{) \begin{array}{l} x \\ 5x^2 - kx + 7 \\ \underline{5x^2 - 2x} \end{array}}$	<p><b>Step 2</b></p> $5x - 2 \overline{) \begin{array}{l} x \\ 5x^2 - kx + 7 \\ \underline{-5x^2 + 2x} \\ x(2-k) + 7 \end{array}}$
<p><b>Step 3</b></p> $5x - 2 \overline{) \begin{array}{l} x + \frac{2-k}{5} \\ 5x^2 - kx + 7 \\ \underline{-5x^2 + 2x} \\ x(2-k) + 7 \\ x(2-k) - \frac{2}{5}(2-k) \end{array}}$	<p><b>Step 4</b></p> $5x - 2 \overline{) \begin{array}{l} x + \frac{2-k}{5} \\ 5x^2 - kx + 7 \\ \underline{-5x^2 + 2x} \\ x(2-k) + 7 \\ \underline{-x(2-k) + \frac{2}{5}(2-k)} \\ 7 + \frac{2}{5}(2-k) \end{array}}$	

1. How many times does  $5x$  go into  $5x^2$ ?  $x$  times. Now write the  $x$  on top of the division and multiply down, like you do in normal division.

2. Now subtract the multiplied term from the divisor and drop down the next number.

3. How many times does  $5x$  go into  $x(2-k)$ ?  $\frac{2-k}{5}$  times. Now write  $\frac{2-k}{5}$  on top of the division and multiply down.

4. Subtract down, and we get a remainder of  $7 + \frac{2}{5}(2-k)$ .

We know the remainder is 6, so we set  $7 + \frac{2}{5}(2-k) = 6$ , and solve for  $k$ . We get  $k = \boxed{\frac{9}{2}}$

5.  $\boxed{0}$

6.  $\boxed{3x^2 + x}$

7.  $\boxed{1}$

8.  $\boxed{3(x+1)^2 + (-10)(x+1) + 4}$

9.  $\boxed{-1}$

10. First, we find the common denominator on the right side:

$$\frac{x^2 - 2x + 5}{(x-2)^2(x^2+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+1}$$



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Problem Set 13.2 - Coefficients

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$$\begin{aligned} &= \frac{A(x^2 + 1)(x - 2)}{(x - 2)^2(x^2 + 1)} + \frac{B(x^2 + 1)}{(x - 2)^2(x^2 + 1)} + \frac{(Cx + D)(x - 2)^2}{(x - 2)^2(x^2 + 1)} \\ &= \frac{A(x^2 + 1)(x - 2) + B(x^2 + 1) + (Cx + D)(x - 2)^2}{(x - 2)^2(x^2 + 1)} \end{aligned}$$

Now we have the same denominator on both sides. We can multiply both sides by the denominator so it cancels entirely. So now we have:

$$x^2 - 2x + 5 = A(x^2 + 1)(x - 2) + B(x^2 + 1) + (Cx + D)(x - 2)^2$$

Multiply out and simplify the right side:

$$\begin{aligned} &A(x^3 - 2x^2 + x - 2) + B(x^2 + 1) + (Cx + D)(x^2 - 4x + 4) \\ &= Ax^3 - 2Ax^2 + Ax - 2A + Bx^2 + B + Cx^3 - 4Cx^2 + 4Cx + Dx^2 - 4Dx + 4D \\ &= x^3(A + C) + x^2(-2A + B - 4C + D) + x(A + 4C - 4D) + (-2A + B + 4D) \end{aligned}$$

So now we know that this equality is true:

$$x^2 - 2x + 5 = x^3(A + C) + x^2(-2A + B - 4C + D) + x(A + 4C - 4D) + (-2A + B + 4D)$$

In order for this to be possible, the coefficients of each  $x$ -term has to match. So we have a system of equations:

$$\begin{aligned} 0x^3 &= x^3(A + C) \Rightarrow 0 = A + C \\ x^2 &= x^2(-2A + B - 4C + D) \Rightarrow 1 = -2A + B - 4C + D \\ -2x &= x(A + 4C - 4D) \Rightarrow -2 = A + 4C - 4D \\ 5 &= -2A + B + 4D \end{aligned}$$

Using substitution and algebra, we can solve the system of equations for  $A$ ,  $B$ ,  $C$ , and  $D$ .

We get  $A = -.4, B = 1, C = .4, D = .8$