Avid Academy for Gifted Youth

Math Olympiad and Problem Solving Programs

E230 Advanced Math Competitions

Problem Set 13.1 - Quadratics

Name: Date:

1. $\boxed{9}$

2. To find the roots, use the quadratic equation: $x = \dfrac{8 \pm \sqrt{64-4(5)(4)}}{8} = \dfrac{8 \pm \sqrt{-16}}{8}$. Now add the roots: $\dfrac{8+\sqrt{-16}}{8} + \dfrac{8-\sqrt{-16}}{8} = \dfrac{16}{8} = 2.$ $\boxed{\text{E}}$

3. If the roots are reciprocal, then their product is 1. So the roots are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, so find their product: $(\frac{-b+\sqrt{b^2-4ac}}{2a})(\frac{-b-\sqrt{b^2-4ac}}{2a}) = \frac{b^2-b^2+4ac}{4a^2} = \frac{c}{a} = 1$. So the roots will be reciprocal if $\boxed{c=a}$.

4. $\boxed{12}$

5. $\boxed{40}$

6. First factor: $\left(\frac{4x-16}{3x-4}\right)^2 + \left(\frac{4x-16}{3x-4}\right) = 16\left(\frac{x-4}{3x-4}\right)^2 + 4\left(\frac{x-4}{3x-4}\right) = 12$, and after we divide out a 4 we get $4\frac{(x-4)^2(3x-4)^2}{+}\frac{x-4}{3x-4} = 3$. Now multiply by $(3x-4)^2$: $4(x-4)^2+(x-4)(3x-4) = 3(3x-4)^2$. Now multiply out and combine like terms, and we arrive at $0 = 5x^2-6x-8$. Then we use the quadratic equation, and we get that $x = \frac{6\pm14}{10}$. The greater value for x is the plus, so $x = \frac{6+14}{10} = \frac{20}{10} = \boxed{2}$

7. Wewrite the expression using polynomial division: $\dfrac{x^2+2x+5}{x-3} = x+5+\dfrac{20}{x-3}$. The expression will be an integer when $x-3$ divides 20. The divisors of 20 are 1, 2, 4, 5, 10, and 20, so the greatest value of x such that $\dfrac{20}{x-3}$ is an integer is $\boxed{23}$

8. Pick a way to factor $21x^2+ax+21$ into two binomals, for instance $(3x+3)(7x+7)$. Then when you multiply it out, you get $21x^2+42x+21$, which is a particular even number. $\boxed{\text{D}}$

9. $\boxed{\text{A}}$

10. $\boxed{\text{B}}$