

1. (a)  $\boxed{\frac{3}{8}}$

(b)  $\boxed{\frac{1}{32}}$

(c)  $\boxed{\frac{13}{20}}$

2.  $\boxed{A}$ . First, let's find all the sequences of games that will result in team B winning the second game and team A winning the series. Our patterns will look like this: ( ) B ( ) ( ) ... A. A must win the last game, because A wins the series. First, let's see what would happen if team B won the first game. Then the first two games would be won by B. Then team B wouldn't be able to win any more games, because then they would win the series. So if B wins the first game, then the sequence must be: BBAAA. Now let's consider the possibilities if A wins the first game. Try to find the combinations yourself, and you will find you get: ABAA, ABABA, ABBA.

Now let's find the likelihood of each sequence of games. The four-game series has probability  $\frac{1}{2} = \frac{1}{16}$ , and any five game series has probability  $\frac{1}{2} = \frac{1}{32}$ . Thus the probability of a four-game series is twice as likely as a five games series. Let  $p$  be the probability of the occurrence of the sequence ABBA. Then the probability of ABABA is also  $p$ , and the probability of BBAAA is  $p$ . The probability of ABAA is  $2p$ . So  $2p + p + p + p = 1$ , and  $p = \frac{1}{5}$ .

3. Let's find the prime factors of 30: 2, 3, 5. Now let's find the numbers that do NOT 2, 3, or 5 in their prime factorization: 1, 7, 11, 13, 17, 19, 23, 29. So there are 8 numbers that do not have any factors of 2, 3, or 5 in them. So if we randomly pick a number from 1 to 30, the probability that it has no common factors with 30 is  $\frac{8}{30} = \boxed{\frac{4}{15}}$

4.  $\boxed{\frac{1}{216}}$

5. Using basic counting, there are  $4!$  ways of arranging four different CDs into four distinct cases. Now let's count how many ways we have exactly two CDs in their correct cases. If the CDs A, B, C, and D need to go in that order, then there are six ways that they can be rearranged so that two are correct and two are incorrect. For instance, the arrangement (A)(B)DC has A and B in the correct positions, but D and C are in the wrong cases. Count out the other cases: (A)D(C)B, (A)CB(D), D(B)(C)A, C(B)A(D), BA(C)(D). So there are six such arrangements, and 24 possible arrangements, so our probability is  $\frac{6}{24} = \boxed{\frac{1}{4}}$

6.  $\boxed{\frac{2}{9}}$

7.  $\boxed{\frac{64}{125}}$

8. First, let's see how many combinations of straw lengths there are. We can pick four possible lengths for the first box, four for the second, and four for the third. So there are  $4^3 = 64$  combinations. Now we count how many ways we can draw three numbers that can be made into a triangle. To do this, I quickly listed all the possible combinations of three numbers, and then crossed out the combinations that could not make triangles. For instance, side lengths of 1, 1, and 2 cannot make a triangle. So we are left with the following sequences of



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side lengths: 111, 222, 333, 444 (there is 1 way to choose these combinations), 122, 133, 144, 223, 233, 244, 334, 344 (there are  $C(3, 2) = 3$  ways to choose these combinations), and 234 (there are  $3! = 6$  ways to choose this combination). So there are  $4 \times 1 + 8 \times 3 + 1 \times 6 = 34$  ways to choose straws that can be made into a triangle. So the probability is  $\frac{34}{64} = \boxed{\frac{17}{32}}$

9.  $\boxed{\frac{5}{9}}$
10.  $\boxed{\frac{5}{9}}$