

1. **E**. Consider the coins as a single set. The value is $1 + 5 + 10 + 25 + 50 = 91$. Then the number of sets is $273 \div 91 = 3$. Each set has 5 coins so the total is $3 \times 5 = \boxed{15}$.
2. **C**. The first boy paid $\frac{1}{3}60 = \$20$. The second boy paid $\frac{1}{4}60 = \$15$. The third boy paid $\frac{1}{5}60 = \$12$. The fourth boy paid $60 - 20 - 15 - 12 = \boxed{\$13}$.
3. **C**. We can set a system of equations

$$\begin{aligned} 3x &= 4y \\ x - y &= 8 \end{aligned}$$

Solve for $x = \boxed{32}$.

4. **D**. The man made a total of $8 + 15 + 9 = 32$ one-way commutes. Thus, $x = \frac{32}{2} = \boxed{16}$ working days.
5. **D**. We can set a system of equations

$$a = b + c + 16 \tag{1}$$

$$a^2 = (b + c)^2 + 1632 \tag{2}$$

Now, we have 3 variables, but only two equations, which means at least 2 variables in the equation are not independent from each other. We also note that the equation (2) can be turned into a difference of squares such that

$$a^2 - (b + c)^2 = (a + b + c)(a - b - c) = 1632.$$

Now, if we move the variables of equation (1) to the left side, we have $a - b - c = 16$. Thus, $a + b + c = \frac{1632}{16} = \boxed{102}$.

Problem Solving Strategy: When you find in a situation of not enough equations to solve variables, it usually means some hidden relationships among the variables given. Then, looking for algebraic manipulations to discover such hidden relationships.

6. **B**. Let a be Ann's age and b be Barbara's age now. When Barbara was half of Ann's age, it was $b - \frac{a}{2}$ years ago. Then, Ann will be $a - (b - \frac{a}{2}) = \frac{3}{2}a - b$ years old. When Barbara was Ann's age then, which is $\frac{3}{2}a - b$, it is $b - (\frac{3}{2}a - b) = 2b - \frac{3}{2}a$ years ago. Then, Ann's age is $a - (2b - \frac{3}{2}a) = \frac{5}{2}a - 2b$ years old. Now, we have the equation:

$$\begin{aligned} a + b &= 44 \\ b &= \frac{5}{2}a - 2b \end{aligned}$$

Solve for a , we have $a = \boxed{24}$.

7. **A**. When two cars travels the same amount of time, their average speed is an arithmetic mean. Thus, $y = \frac{u+v}{2}$. When the to cars travels the same amount of distance, their average speed is a harmonic mean. Thus, $x = \frac{2}{\frac{1}{u} + \frac{1}{v}}$. Thus, we have $\boxed{x \leq y}$ where equality holds if $u = v$.



Math Olympiad and Problem Solving Programs

E230 - Advanced Math Competitions

Grades 7 - 8

Problem Set 5.2 - Word Problems

Name:

Date:

8. (AHSME 73-34) A plane flew straight against a wind between two towns in 84 minutes and returned with that wind in 9 minutes less than it would take in still air. The number of minutes for the returned trip was.
- (A) 54 or 18 (B) 60 or 15 (C) 63 or 12 (D) 72 or 36 (E) 75 or 20
9. (AHSME 73-33) When one ounce of water is added to a mixture of acid and water, the new mixture is 20% acid. When one ounce of acid is added to the new mixture, the result is $33\frac{1}{3}\%$ acid. The percentage of acid in the original mixture
- (A) 22% (B) 24% (C) 25% (D) 30% (E) $33\frac{1}{2}$