Avid Academy
for Gifted Youth

Math Olympiad and Problem Solving Programs

E230 - Advanced Math Competitions

Grades 7 - 8

Problem Set 4.1 - Combinations Solutions

Name: Date:

1. $\boxed{30}$. We make separate selections for boys and girls and then use the product rule to combine them. There are $\binom{3}{2} = 3$ ways to select the girls and $\binom{5}{2} = 10$ ways to select the boys. Therefore, the total is $3 \times 10 = \boxed{30}$.

2. $\boxed{840}$. Similar to last problem, we have $\binom{8}{3} = 56$ ways to select boys and $\binom{6}{2} = 15$ ways to select the girls. The total selection is $56 \times 15 = \boxed{840}$ ways.

3. $\boxed{41472}$. In a band, we assume (a very logical assumption) each player knows which section they belong based on the instrument he or she play. Then, there $4!3!3!2 = 1728$ ways to seat the students. However, there is also a need to arrange how the sections are organized. There are $4! = 24$ ways to organize the sections. Therefore, the total seating arrangement is $1728 \times 24 = \boxed{41,472}$.

   Now, let's assume this is the first class of a beginner's band and the band director wants to assign students the instruments before seating them. For simplicity, we assume the band sections are already determined. Then, we can select each group in order and then count the number of arrangements within each group. Thus, there are $(\binom{12}{4} \cdot 4!)(\binom{8}{3} \cdot 3!)(\binom{5}{3} \cdot 3!)(\binom{2}{2} \cdot 2!) = 12!$ ways. Is it interesting that the answer is the same as seating 12 people in a single row? What will be the answer if we need to consider different ways to configure the band sections?

   If the question is about how many ways to select people to each group, but not the ways to seat them, then there are $\binom{12}{4}\binom{8}{3}\binom{5}{3}\binom{2}{2} = \frac{12!}{4!3!3!2!}$. Did you see the difference between permutation and combination?

4. $\boxed{18{,}816}$. This problem involves both selection and ordering. First, we need to select 2 of the 8 contestants to the painting group and then select 3 of from the 8 contestants to the singing group. Within each group, we need arrange the placements. Thus, we have $(\binom{8}{2} \cdot 2!)(\binom{8}{3}3!) = (8 \times 7)(8 \times 7 \times 6) = \boxed{18,816}$.

5. $\boxed{28}$. There are $\binom{8}{2} = \boxed{28}$ ways.

6. $\boxed{5}$. Let 45 be the sum of n consecutive integers (positive or negative). If n is odd and the medium of the n consecutive integer is x, Then, we can express $nx = 45$ where n is the number of consecutive integers and x is the medium of the integers. Since $45 = 3^2 \times 5$ and have six factors, there six ways to assign value to n and x. Of the six, $n = 1$ and $x = 45$ should be excluded because it is only one integer. If $n = 15$, the corresponding sequence is $-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, which is equivalent to $5, 6, 7, 8, 9, 10$ that sum to 45. Similar for $n = 45$ which will yield $22, 23$. Therefore, we have a total of $\boxed{5}$ ways.

7. $\boxed{210}$. A quadrilateral is formed by connecting 4 points. The order of selecting the points are not important. Therefore, we have $\binom{10}{4} = \boxed{210}$ ways.

   Remark: Such a quadrilateral is called cyclic quadrilateral. In a cyclic quadrilateral, the sum of opposite angles is $180°$.

8. $\boxed{120}$. Let X be the place where a car goes and O be an empty spot, we have the following possible configurations:

   XOXOXOXO XOOXOXOX XOXOOXOX XOXOXOOX OXOXOXOX

   Therefore, the total selection is $5 \times 4! = \boxed{120}$.

9. $\boxed{36}$. Let's define a gap being the space between two adjacent letters where a slash can be inserted, the question became how many ways to place two slashes on 9 gaps without considering the order of selection. Thus, the answer is $\binom{9}{2} = \boxed{36}$.

10. $\boxed{180}$. The number of ways is $\binom{3}{1} \times \binom{4}{2} \times \binom{5}{2} = \boxed{180}$.