Math Olympiad and Problem Solving Programs
E230 - Advanced Math Competitions
Problem Set 2.2 - Counting Numbers

Name: Date:

1. Find the numbers that have a factor of 3 in 70!: $3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 18 \cdot 21 \cdot 24 \cdot 27 \cdot 30 \cdot 33 \cdot 36 \cdot 39 \cdot 42 \cdot 45 \cdot 48 \cdot 51 \cdot 54 \cdot 57 \cdot 60 \cdot 63 \cdot 66 \cdot 69$. Now count the factors of 3's: $1+1+2+1+1+2+1+1+3+1+1+2+1+1+2+1+1+3+1+1+2+1+1 = \boxed{32}$

2. The digit 2 is used from numbers 1 - 99 20 times, and the digit 2 is used from numbers 100 - 199 another 20 times. Now consider 200 - 209. The digit 2 is used 11 times. From 210 - 219, 11 times. There are 44 2's left to go. From 220 - 229, 2 is used 21 times. 23 2's to go. From 230 - 239, 11 2's. From 240 - 249, 11 2's. Just 1 2 to go. So the last locker is $\boxed{250}$.

3. $5 \times 4 \times 4 = \boxed{80}$

4. $\boxed{21}$

5. The possible numbers are $2+3+19$, $2+7+17$, and $2+11+11$. There are $3! = 6$ ordered triples (i.e., a set of three numbers where order matters) for 2, 3, and 17, $3! = 6$ for 2, 7, and 17, and $3! \div 2 = 3$ for 2, 11, and 11. So there are $6+6+3 = \boxed{15}$ ordered triples.

6. The first digit must be 1, and there are three digits. So there are 10 numbers of the form $11x$, where x is any digit 0-9. There are 10 numbers of the form $1xx$, where x is any digit 0-9. But we over counted by one: both of our counting methods include the number 111. So there are 100 numbers between 100-199 inclusive, and $10+10-1 = 19$ digits with consecutive identical digits. So there are $100 - 19 = \boxed{81}$ without consecutive identical digits.

7. $\boxed{15}$

8. $\boxed{176}$

9. $\boxed{D}$

10. $\boxed{B}$