



Math Olympiad and Problem Solving Programs
E230 - Advanced Math Competitions
Problem Set 2.1 - Permutations

Name:

Date:

- $5 \times 4 = \boxed{20}$
- There are three distinct consonants (R, N, G) and two distinct vowels (E, I). So you can choose one of three consonants and one of two vowels, so: $3 \times 2 = \boxed{6}$
- There are $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$ six-digit numbers of odd digits, and there are $4 \times 5 \times 5 \times 5 \times 5 \times 5 = 4 \times 5^5$ six-digit numbers of all even digits. We add $5 \times 5^5 + 4 \times 5^5 = 9 \times 5^5 = \boxed{9 \times 5^5 = 28125}$
- $20^{30} = (2^2 \times 5)^{30} = 2^{60} \times 5^{30}$. Now take each exponent and add one, and multiply the resulting numbers: $(30 + 1)(60 + 1) = 31 \times 61 = \boxed{1891}$.
- $4 \times 10 \times 1 = \boxed{40}$
- $\frac{7(7-3)}{2} = 7 \times 2 = \boxed{14}$
- Numbers between 3000 and 8000 will be four digits long and start with either 3, 4, 5, 6, or 7. So I set up a picture where I will write the number of possibilities on each line.

~~(3)~~ _ _ _
~~(4)~~ _ _ _
~~(5)~~ _ _ _
~~(6)~~ _ _ _
~~(7)~~ _ _ _

Consider the last digit. It must be an odd digit since the number is an odd number. Thus, when the first digit is 4 or 6, then there are still five possible odd digits to be used for the last digit. When the first digit is a 3, 5, or 7, there are only 4 more possible odd digits left for the last digit. Write these numbers in.

~~(3)~~ _ _ 4
~~(4)~~ _ _ 5
~~(5)~~ _ _ 4
~~(6)~~ _ _ 5
~~(7)~~ _ _ 4

Now lets consider the middle two digits. Since no digit can be repeated, we now have 8 possibilities for the third digit (since two have been used on the first and last digit), and 7 possibilities for the fourth digit. Fill this in.

~~(3)~~ 8 7 4
~~(4)~~ 8 7 5
~~(5)~~ 8 7 4
~~(6)~~ 8 7 5
~~(7)~~ 8 7 4

So there are $(1 \times 8 \times 7 \times 4) + (1 \times 8 \times 7 \times 5) + (1 \times 8 \times 7 \times 4) + (1 \times 8 \times 7 \times 5) + (1 \times 8 \times 7 \times 4) = 3(8 \times 7 \times 4) + 2(8 \times 7 \times 5) = \boxed{1232}$.

- (a) $\boxed{11!}$
(b) $\boxed{\frac{7!6!}{12}}$
(c) $\boxed{6!5!}$



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9. E
10. A