

1.  $|A \cap B| = \boxed{2}$ ,  $|A \cap C| = \boxed{0}$ ,  $|A \cup B| = \boxed{5}$ , and  $|A \cup C| = \boxed{6}$ .

2. (a)  $(A \cap B) \cap C = A \cap (B \cap C) = \boxed{\{2\}}$

(b)  $(A \cup B) \cup C = A \cup (B \cup C) = \boxed{\{1, 2, 3, 4, 5, 6, 7, 8, 11\}}$

(c)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \boxed{\{2, 4, 6\}}$

(d)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \boxed{\{2, 3, 4, 5, 6, 8\}}$

3. (a)  $|A| = \boxed{10}$  (b)  $|B| = \boxed{6}$  (c)  $|A \cup B| = \boxed{13}$  (d)  $|A \cap B| = \boxed{3}$

4. (a)  $\boxed{5}$

(b)  $\boxed{6}$

5. The  $\subseteq$  symbol means subset; for instance,  $\{3, 5\}$  is a subset of  $\{1, 3, 5, 7\}$ , so we write  $\{3, 5\} \subseteq \{1, 3, 5, 7\}$ . The set  $X$  must be a subset of  $\{1, 2, 3, 4, 5\}$  and the set  $\{1, 2\}$  is a subset of  $X$ . There are 8 possible subsets:  $\{1, 2\}$ ,  $\{1, 2, 3\}$ ,  $\{1, 2, 4\}$ ,  $\{1, 2, 5\}$ ,  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 3, 5\}$ ,  $\{1, 2, 4, 5\}$ ,  $\{1, 2, 3, 4, 5\}$ . The answer is  $\boxed{D}$ .

6.  $\boxed{E}$

7. First, *pairwise disjoint sets* means that everytime we pair up two sets, each of them are disjoint, or they have nothing in common. So  $X$  and  $Y$  have nothing in common,  $X$  and  $Z$  have nothing in common, and  $Y$  and  $Z$  have nothing in common.

Next, I will assign variables to the sizes of the sets. Let  $|X| = x$ ,  $|Y| = y$ , and  $|Z| = z$ . Then  $|X \cup Y| = x + y$ ,  $|X \cup Z| = x + z$ , and  $|Y \cup Z| = y + z$ . Since the sets have nothing in common, we can just add their sizes.

The given information can be summarized in the following 3 equations in 3 unknowns:

$$\frac{37x + 23y}{x + y} = 29 \quad \frac{37x + 41z}{x + z} = 39.5 \quad \frac{23y + 41z}{y + z} = 33.$$

Simplifying these equations, we obtain  $4x = 3y$ ,  $5x = 3z$ ,  $5y = 4z$ . We want the value of the fraction  $\frac{37x + 23y + 41z}{x + y + z}$ . Making the substitutions  $y = \frac{4x}{3}$  and  $z = \frac{5x}{3}$ , we obtain

$$\frac{37x + 23\left(\frac{4x}{3}\right) + 41\left(\frac{5x}{3}\right)}{x + \frac{4x}{3} + \frac{5x}{3}} = \frac{111x + 92x + 205x}{3x + 4x + 5x} = 34. \quad \boxed{E}$$

8.  $\boxed{B = D}$

9.  $\boxed{5}$

10.  $\boxed{B}$