



Math Olympiad and Problem Solving Programs
E220 - Intermediate Math Competitions
Problem Set 27.2 - Modular Residues

Name:

Date:

Instruction: Calculate the following modulo RESIDUES. This means that your answers cannot be negative or greater than the mod.

1. Since $6 \times 15 = 90$, write $101 = 90 + 11$, or $-101 = -90 - 11$. So $-101 \pmod{15} \equiv -90 - 11 \equiv 0 - 11 \equiv \boxed{4}$
2. $41 \pmod{9} \equiv \boxed{5}$
3. $32649 \pmod{16} \equiv \boxed{9}$
4. $26 + 18 + 14 + 15 + 27 \pmod{28} \equiv -2 - 10 + 14 + 15 - 1 \equiv 2 + 14 \equiv \boxed{16}$
5. $98 + 15 + 11 + 97 + 5 \pmod{101} \equiv -3 + 15 + 11 - 4 + 5 \equiv 12 + 12 \equiv \boxed{24}$
6. $2 \times 60 + 59 \times 59 \pmod{62} \equiv 2 \times -2 + (-3) \times (-3) \equiv -4 + 9 \equiv \boxed{5}$
7. Note: the sum of the integers $1 + 2 + 3 + 4 + \dots + n$ is $\frac{n(n+1)}{2}$. So $1 + 2 + 3 + \dots + 18 + 19 + 20 = \frac{20(21)}{2} = 10 \cdot 21 \pmod{21} \equiv \boxed{0}$
8. $20 \times 28 \pmod{30} \equiv 20 \times -2 \equiv -40 \equiv \boxed{20}$
9. $58 \times 59 \times 60 \pmod{61} \equiv -3 \times -2 \times -1 \equiv -6 \equiv \boxed{55}$
10. $3091 \times 3093 \pmod{3095} \equiv -4 \times -2 \equiv \boxed{8}$
11. $15 \times 12 \times 7 \times 29 \times 8 \pmod{31} \equiv (15 \times 7)(12 \times 8) \times 29 \equiv 105 \times 96 \times -2 \equiv 12 \times 3 \times -2 \equiv 36 \times -2 \equiv 5 \times -2 \equiv -10 \equiv \boxed{21}$
12. $17 \times 12 + 12 \times 12 \pmod{30} \equiv 12(17 + 12) \equiv 12(29) \equiv 12 \times -1 \equiv -12 \equiv \boxed{18}$
13. $187 \times 185 \times 4 \pmod{188} \equiv -1 \times -3 \times 4 \equiv \boxed{12}$
14. $83 \times 10 + 50 \pmod{99} \equiv 10(83 + 5) \equiv 10 \cdot 88 \equiv 10 \cdot -11 \equiv -110 \equiv -11 \equiv \boxed{88}$
15. $23 \times 24 \pmod{47} \equiv (23 \times 4) \times 6 \equiv 92 \times 6 \equiv -2 \times 6 \equiv -12 \equiv \boxed{35}$