

1. $\boxed{2}$

2. $\boxed{7}$

3. Find $4^{18} \pmod{9}$.

$4 \equiv 4 \pmod{9}$. $4^2 = 16 \equiv 7 \pmod{9}$. $4^3 = 64 \equiv 1 \pmod{9}$. So the pattern for mods is 4, 7, 1. So 4^{18} will be 1 (mod 9).

Find $19^{80} \pmod{9}$.

$19 \equiv 1 \pmod{9}$. $19^{80} \equiv 1^{80} \pmod{9} \equiv 1 \pmod{9}$. So 19^{80} will be 1 (mod 9).

So $4^{18} \cdot 19^{80} \pmod{9} \equiv 1 \cdot 1 \pmod{9} \equiv 1 \pmod{9}$. So the remainder will be $\boxed{1}$.

4. $\boxed{1}$

5. The problem asks for TENS AND UNITS digits, not just units digits.

The pattern for tens and units digits is as follows:

$$7^1 \equiv 07 \pmod{100}. \quad 7^2 \equiv 49 \pmod{100}. \quad 7^3 \equiv 43 \pmod{100}. \quad 7^4 \equiv 01 \pmod{100}$$

I used (mod 100) because I only want the last two digits. I see my pattern is 07, 49, 43, 01. We want the 2010th number in the pattern, so we want to find $2010 \pmod{4} = 2$. So the tens and units digits of 7^{2010} is the 2nd number in the pattern, or $\boxed{49}$.

6. (a) $\boxed{2}$ (b) $\boxed{5}$ (c) $\boxed{1}$ (d) $\boxed{1}$

7. (a) $\boxed{4}$ (b) $\boxed{1}$

(c) Observe $6^2 \equiv 36 \pmod{37} \equiv -1 \pmod{37}$. We know $6^{123} = 6^{120+3} = 6^{120} \cdot 6^3 = (6^2)^{60} 6^3$. So then, because $6^2 \equiv -1 \pmod{37}$, then $(6^2)^{60} \equiv (-1)^{60} \pmod{37} \equiv 1 \pmod{37}$. Also, $6^3 = 216 \equiv 31 \pmod{37}$.

So we have $6^{123} \pmod{37} \equiv (6^2)^{60} 6^3 \pmod{37} \equiv 1 \cdot 31 \pmod{37} \equiv \boxed{31}$

(d) Find the pattern of mods.

$$7^1 \equiv 7 \pmod{9}.$$

$$7^2 \equiv 4 \pmod{9}.$$

$$7^3 = 343 \equiv 1 \pmod{9}.$$

The pattern of mods is 7, 4, 1. We have that $777 \div 3 = 259$, or $777 \pmod{3} \equiv 0$. So since the pattern is divisible by 3, it completes 259 complete cycles through the pattern of mods. So $7^{777} \pmod{9} \equiv \boxed{1}$

8. $\boxed{1}$

9. Numbers that are 3 (mod 7) are of the form $7k + 3$. We want to find all such numbers such that $100 \leq 7k + 3 \leq 999$. Subtract all sides by 3: $97 \leq 7k \leq 996$. Now divide all sides by 7: $13.8 \leq k \leq 142.3$. So for values of k in the list 14, 15, 16, ..., 142, we have a three digit number that is 3 (mod 7). There are $142 - 14 + 1 = \boxed{129}$ values in this list.



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10. Since $36 \equiv 2 \pmod{17}$, let's simplify finding $36^{2010} \pmod{17}$ by finding $2^{2010} \pmod{17}$. Notice that $2^4 = 16 \equiv -1 \pmod{17}$. So let's write $2^{2010} = 2^{2008} \cdot 2^2 = (2^4)^{502} \cdot 2^2 \equiv (-1)^{502} \cdot 2^2 \pmod{17} \equiv 4$. So $36^{2010} \equiv 4 \pmod{17}$.

Now let's look at $15^{2009} \pmod{17}$. Notice $15^{2009} \equiv 16 \pmod{17} \equiv -1 \pmod{17}$. We write $15^{2009} = 15^{2008} \cdot 15 = (15^4)^{502} \cdot 15 \equiv (-1)^{502} \cdot 15 \pmod{17} = 1 \cdot 15 \pmod{17} = 15$. So $15^{2009} \pmod{17} = 15$.

So finally, $36^{2010} + 15^{2009} \pmod{17} \equiv 4 + 15 \pmod{17} \equiv 19 \pmod{17} \equiv \boxed{2}$