1. $5 \times 4 = \boxed{20}$

2. The problem implies that order doesn't matter. So there are 4 consonants to choose from, C, D, M, Y. There are 2 vowels to choose from, A, E. So there are $4 \times 2 = \boxed{8}$ ways to choose.

3. In a six-digit number, the second, third, fourth, fifth, and last digits have 5 choices: 0,2,4,6,8. The first digit only has 4 choices, because the first digit cannot be zero. So there are $4 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = \boxed{4 \cdot 5^5 = 12500}$ such numbers.

4. How many ways to choose a card? **52 ways**. Say we choose a 3 of diamonds. Now we take out all the other diamonds (12 cards) and the other 3's (3 cards) from the deck, and we have 36 cards remaining. Now we choose a second card, out of **36 cards**. Say we choose a 7 of hearts. Now we take the remaining hearts (11 cards) and the remaining 7's (2 cards) out of the deck, and we have **22 cards**. Then we choose another card, take out the remaining cards with the same suit and number, and then we have **10 cards** left to choose from. So there are $\boxed{52 \cdot 36 \cdot 22 \cdot 10 = 411840}$ ways to choose four such cards.

   The problem does not specify if order matters. If order does matter, use the above answer. If it does not matter, we divide out by 4!, and we get $\boxed{17160}$.

5. We use permutations because the stack of books ABC is different from the stack ACB, so order matters.

   How many ways to arrange one book in a stack? $P_1^5 = \frac{5!}{4!} = 5$.

   How many ways to arrange two books in a stack? $P_2^5 = \frac{5!}{3!} = 20$.

   How many ways to arrange three books in a stack? $P_3^5 = \frac{5!}{2!} = 60$.

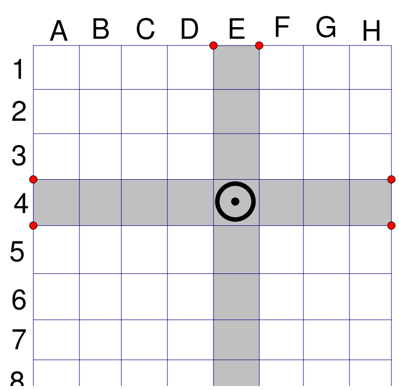
   How many ways to arrange four books in a stack? $P_4^5 = \frac{5!}{1!} = 120$.

   How many ways to arrange all five books in a stack? $P_5^5 = \frac{5!}{0!} = 120$.

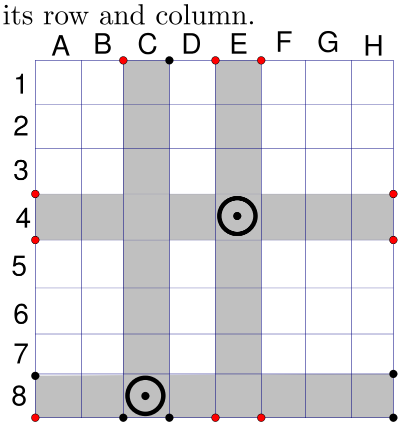
   Add up all the arrangements, and we get $\boxed{325}$.

6. Rooks move in straight lines on the chessboard. So we need to make sure that none are in the same direct line from each other. Also, note that chessboards have labeled squares, so each square is unique on a chessboard.

   Let's place our first rook. There are 64 squares to choose from. Now cross out the row and column the rook is in.



Now there are 49 spaces to choose from on our chessboard. So select a random square for the second rook and cross out its row and column.



Now there are 36 squares available for the third rook. Continue placing rooks randomly and counting how many squares are available for placing rooks.

We find that there are $64 \cdot 49 \cdot 36 \cdot 25 \cdot 16 \cdot 9 \cdot 4 \cdot 1$ combinations. Rewrite this: $8^2 \cdot 7^2 \cdot 6^2 \cdot 5^2 \cdot 4^2 \cdot 3^2 \cdot 2^2 \cdot 1^2 = (8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)^2 = \boxed{(8!)^2}$

7. $\boxed{\binom{18}{2} = 153}$

8. We want to know how many ways Brian can arrange the letters AAPPPOOOO. There are 9! ways of doing this, but we need to divide out the overcounting. The method for doing these problems (for the next two problems as well), you divide out as many repeats factorial. For instance, we have 2 A's, so we divide out 2!. We have 3 P's, so we also divide out 3!. There are 4 O's, so we also divide out 4!. So our answer is $\boxed{\frac{9!}{2!3!4!} = 1260}$

9. Method 1: as explained in the problem before, we take the factorial of how many things there are (7 students) and divide by each repeat factorial (2 repeats and 4 repeats). This gives us $\boxed{\frac{7!}{4!2!} = 105}$.

   Method 2: First, let's choose the four students to go in the 4-person dorm. Since order doesn't matter, we use combinations. $\binom{7}{4} = \frac{7!}{3!4!}$. Now let's choose the students who will go in the double room; the remaining student will go in the single. So from our 3 students left, we choose 2: $\binom{3}{2} = \frac{3!}{2!1!}$. Now we multiply together and get $\frac{7!}{3!4!} \cdot \frac{3!}{2!} = \boxed{\dfrac{7!}{4!2!} = 105}$.

10. Use the method described in problem 8. First we have 8! ways of arrangeing the figures. Now we divide out the repeats. There are 2 identical rooks, so we divide out 2!, 2 identical knights, so we divide out 2!, and 2 identical bishops, so we divide out 2! a third time. So we have $\frac{8!}{2!2!2!} = \frac{8!}{8} = \boxed{7! = 5040}$