1.

$$\begin{array}{r} 3x^2 \quad +6x \quad +7 \\ x-2\overline{\big)\ 3x^3 \qquad\quad -5x \quad +6} \\ \underline{-3x^3+6x^2} \qquad\qquad\quad \\ 6x^2 \quad -5x \qquad\quad \\ \underline{-6x^2+12x} \qquad\quad \\ 7x \quad +6 \\ \underline{-7x+14} \\ 20 \end{array}$$

$\boxed{Q = 3x^2 + 6x + 7, R = 20}$

2.

$$\begin{array}{r} 2x^2 \quad -x-1 \\ x+3\overline{\big)\ 2x^3+5x^2-4x+8} \\ \underline{-2x^3-6x^2} \qquad\qquad \\ -x^2-4x \qquad \\ \underline{x^2+3x} \qquad \\ -x+8 \\ \underline{x+3} \\ 11 \end{array}$$

$\boxed{Q = 2x^2 - x - 1, R = 11}$

3.

$$\begin{array}{r} 3x^2 \quad -7x+5 \\ 2x-1\overline{\big)\ 6x^3-17x^2+17x-5} \\ \underline{-6x^3 \quad +3x^2} \qquad\qquad \\ -14x^2+17x \qquad \\ \underline{14x^2 \quad -7x} \qquad \\ 10x-5 \\ \underline{-10x+5} \\ 0 \end{array}$$

$\boxed{3x^2 - 7x + 5}$

4.

$$\begin{array}{r} 9x^2 \quad +3x+\frac{11}{3} \\ 3x-2\overline{\big)\ 27x^3 \quad -9x^2 \quad +5x \quad -2} \\ \underline{-27x^3+18x^2} \qquad\qquad\qquad \\ 9x^2 \quad +5x \qquad\quad \\ \underline{-9x^2 \quad +6x} \qquad\quad \\ 11x \quad -2 \\ \underline{-11x+\frac{22}{3}} \\ \frac{16}{3} \end{array}$$

$\boxed{Q = 9x^2 + 3x + \frac{11}{3}, R = \frac{16}{3}}$

5.

$$
\begin{array}{r|l}
 & 6x^4 - 6x^3 + 2x^2 + 3x \\ \hline
x+1 & 6x^5 \quad - 4x^3 + 5x^2 + 3x + 8 \\
 & -6x^5 - 6x^4 \\ \cline{2-2}
 & \quad -6x^4 - 4x^3 \\
 & \quad 6x^4 + 6x^3 \\ \cline{2-2}
 & \quad\quad 2x^3 + 5x^2 \\
 & \quad\quad -2x^3 - 2x^2 \\ \cline{2-2}
 & \quad\quad\quad 3x^2 + 3x \\
 & \quad\quad\quad -3x^2 - 3x \\ \cline{2-2}
\end{array}
$$

$\boxed{8}$

6.

$$
\begin{array}{r|l}
 & x^3 \quad + 8x \quad + (-24 + -1k) \\ \hline
x+3 & x^4 + 3x^3 + 8x^2 \quad - kx \quad + 11 \\
 & -x^4 - 3x^3 \\ \cline{2-2}
 & \quad 8x^2 \quad - kx \\
 & \quad -8x^2 \quad - 24x \\ \cline{2-2}
 & \quad\quad (-24 + -1k)\,x \quad + 11 \\
 & \quad\quad -(-24 + -1k)\,x \quad -3(-24 + -1k) \\ \cline{2-2}
 & \quad\quad\quad (-13 + -1 + -3k)
\end{array}
$$

Notice the last line of the division is incorrect. (I can't fix it manually; the division is done by the computer). $11 - 3(-24 - k) = 83 + 3k$, not $-14 - 3k$. Since we want it to evenly divide, the remainder must be 0. So we find where $83 + 3k = 0 \Rightarrow 3k = -83 \Rightarrow \boxed{k = -\frac{83}{3}}$

7.

$$
\begin{array}{r|l}
 & x^2 \quad + \tfrac{1}{4}x \quad + \tfrac{1}{2}\left(\tfrac{-1}{8} + 1k\right) \\ \hline
2x + \tfrac{1}{2} & 2x^3 \ + x^2 \quad + kx \quad - 2 \\
 & -2x^3 - \tfrac{1}{2}x^2 \\ \cline{2-2}
 & \quad \tfrac{1}{2}x^2 \quad + kx \\
 & \quad -\tfrac{1}{2}x^2 \quad - \tfrac{1}{8}x \\ \cline{2-2}
 & \quad\quad \left(\tfrac{-1}{8} + 1k\right)x \quad - 2 \\
 & \quad\quad -\left(\tfrac{-1}{8} + 1k\right)x \quad - \tfrac{1}{4}\left(\tfrac{-1}{8} + 1k\right) \\ \cline{2-2}
 & \quad\quad\quad \left(\tfrac{-17}{8} + 1 + \tfrac{-1}{4}k\right)
\end{array}
$$

Notice the last line of the division is incorrect. (I can't fix it manually; the division is done by the computer). $-2 - \frac{1}{4}(-\frac{1}{8} + k) = -\frac{63}{32} - \frac{k}{4}$, not what is shown. Since we want it to evenly divide, the remainder must be 0. So we find where $-\frac{63}{32} - \frac{k}{4} = 0 \Rightarrow \frac{k}{4} = -\frac{63}{32} \Rightarrow \boxed{k = -\frac{63}{8}}$

8.

$$\begin{aligned} f(-\frac{1}{3}) &= 3(-\frac{1}{3})^5 - 17(-\frac{1}{3})^4 + 12(-\frac{1}{3})^3 + 6(-\frac{1}{3})^2 + 9(-\frac{1}{3}) + 8 \\ &= -\frac{3}{3^5} - \frac{17}{3^4} - \frac{12}{3^3} + \frac{6}{3^2} - \frac{9}{3} + 8 \\ &= -\frac{1}{3^4} - \frac{17}{3^4} - \frac{4}{3^2} + \frac{6}{3^2} - 3 + 8 \\ &= -\frac{18}{3^4} + \frac{2}{3^2} + 5 \\ &= -\frac{2}{9} + \frac{2}{9} + 5 \\ &= \boxed{5} \end{aligned}$$

9. $(x+1)(x+2) = x^2 + 3x + 2$. When we long divide $x^2 + 3x + 2$ into $x^4 - ax^2 - bx + 2$, you end up with the remainder $x[(6-b) - 3(7-a)] + [2 - 2(7-a)]$. Since the remainder must be zero, the coefficient on the x term must be zero and the last coefficient must also be zero.

$$\begin{aligned} 2 - 2(7-a) &= 0 \\ 2 - 14 + 2a &= 0 \\ -12 + 2a &= 0 \\ 2a &= 12 \\ a &= \boxed{6} \\ (6-b) - 3(7-a) &= 0 \\ 6 - b - 21 + 3a &= 0 \\ -15 - b + 3(6) &= 0 \\ 3 - b &= 0 \\ b &= \boxed{3} \end{aligned}$$

10.

$$\begin{array}{r@{}l} & \tfrac{3}{2}x^3 - x^2 + \tfrac{1}{2}x + \tfrac{1}{2} \\ 2x-4 \big) & \overline{3x^4 - 8x^3 + 5x^2 - x + 8} \\ & \underline{-3x^4 + 6x^3} \\ & \quad -2x^3 + 5x^2 \\ & \quad \underline{2x^3 - 4x^2} \\ & \qquad x^2 - x \\ & \qquad \underline{-x^2 + 2x} \\ & \qquad\quad x + 8 \\ & \qquad\quad \underline{-x + 2} \\ & \qquad\qquad 10 \end{array}$$

$\boxed{10}$