1. $\boxed{m = 2, n = -15}$

2. $\boxed{f(3) = 25}$

3. $\boxed{g(x) = x^2 + 6x + 2}$

4. $\boxed{f(x) = -\frac{5}{2}x^2 + \frac{11}{2}x + 1}$

5. $\boxed{a = -1, b = -1, c = 1}$

6. Foil out $(x^2 + mx + n)^2 = x^4 + mx^3 + nx^2 + mx^3 + m^2x^2 + nmx + nx^2 + nmx + n^2 = x^4 + 2mx^3 + (2n + m^2)x^2 + 2nmx + n^2$. Set it equal to $x^4 - 6x^3 + 13x^2 - 12x + k$, and matching up the coefficients on each of the x terms, we see that $2m = -6$, $2n + m^2 = 13$, $2nm = -12$, and $n^2 = k$. So from the first equation we see that $m = -3$. Now we can apply the third equation, which says $2nm = -12$, and plug in -3 for m, and we get $-6n = -12$, so $n = 2$. Now we can get k from the last equation, which is $n^2 = 2^2 = 4 = k$. $\boxed{m = -3, n = 2, k = 4}$

7. $\boxed{m = -10, k = -5, l = 2}$

8. First, foil and distribute out everything in $a(x+1)^2+b(x+1)+c$ and get $ax^2+2ax+a+bx+b+c$, or $ax^2 + x(2a + b) + (a + b + c)$. Set each coefficient of each x term equal to each other: $a = 3$, $2a + b = -4$, $a + b + c = 7$. Since $a = 3$, $2(3) + b = -4$ so $b = -10$. Then $a + b + c = 7 = 3 - 10 + c = 7$, so $c = 14$. So we write the polynomial as $\boxed{3(x + 1)^2 - 10(x + 1) + 14}$

9. First, add the fractions together by finding the common denominator, which is $(x+2)(x+3)$.
$$\frac{A}{x+2} + \frac{B}{x+3} = \frac{A(x+3)}{(x+2)(x+3)} + \frac{B(x+2)}{(x+3)(x+2)} = \frac{A(x+3) + B(x+2)}{(x+3)(x+2)}$$
So we know that $\dfrac{x+4}{x^2+5x+6} \equiv \dfrac{A(x+3) + B(x+2)}{(x+3)(x+2)}$, so since the denominators are the same, the numerators must be the same. So $x + 4 \equiv A(x + 3) + B(x + 2)$. Distribute out the right side of the equation: $Ax + 3A + Bx + 2A = x(A + B) + (3A + 2B)$. So we match the coefficients on each term and we find $1 = A + B$ and $4 = 3A + 2B$. Using substitution or equation addition, we find that $\boxed{A = 2, B = -1}$.

10. $\boxed{a = 6, b = \frac{15}{2}}$