



Math Olympiad and Problem Solving Programs  
E220 - Intermediate Math Competitions  
Problem Set 18.2 - Substitution

Name:

Date:

**Instruction:** Solve each quadratic equation by substitution. Show your work. Note: Problems 8-10 have four solutions!

1.  $x = 1, 4$

2.  $x = 10, -2$

3.  $x = 125, -27$

4.  $y = 1, \frac{1}{2}$

5. Let  $u = \sqrt{z}$ . Then  $u^2 = z$ . Substitute, and we have  $u^2 - 9u + 14 = 0$ . This factors to  $(u - 7)(u - 2) = 0$ , so  $u = 2, 7$ . Substituting back in for  $z$ ,  $\text{sqrt}z = 2, 7$ , so squaring both sides gives us  $z = 49, 4$

6.  $x = \frac{1}{16}$

7. Let  $u = x^{-1}$ . Then  $u^2 = x^{-2}$ . Substitute, and we have  $10u^2 + 3u - 1 = 0$ . Using the quadratic equation, we get  $u = \frac{-3 \pm \sqrt{9 - 4(10)(-1)}}{2 \cdot 10} = \frac{-3 \pm \sqrt{49}}{20} = \frac{-3 \pm 7}{20}$ . So  $u = \frac{-3 + 7}{20} = \frac{4}{20} = \frac{1}{5}$  or  $u = \frac{-3 - 7}{20} = \frac{-10}{20} = -\frac{1}{2}$ . Substituting back in for  $x$ ,  $x^{-1} = \frac{1}{5}, -\frac{1}{2}$ , so raising both sides to the  $-1$  power gives us  $x = 5, -2$

8.  $y = \pm\frac{3}{2}, \pm 1$

9.  $x = \pm 2, \pm\sqrt{3}$

10.  $t^4 - 4 = 0$   $x = \pm\sqrt{2}, \pm i\sqrt{2}$