

1. We want $x^2 + kx + \frac{1}{9} = (x \pm \frac{1}{3})^2$, and factoring this out gives us $k = \pm \frac{2}{3}$

2. (a) $(x + 7)^2$

(b) $(x - 11)^2$

(c) $(x + 6)^2$

3. Recall: $a^2 - b^2 = (a + b)(a - b)$.

(a) $(2m + 7n)(2m - 7n)$

(b) $75r^4 - 27g^2 = 3(25r^4 - 9g^2) = 3((5r^2)^2 - (3g)^2) = 3(5r^2 + 3g)(5r^2 - 3g)$

4. First, notice that $+(b - a) = -(a - b)$. So we can replace the $b - a$ in the equation with $-(a - b)$:

$$\begin{aligned} (a - b)a^2 + (b - a)b^2 &= (a - b)a^2 - (a - b)b^2 \text{ (factor out } a - b) \\ &= (a - b)(a^2 - b^2) \text{ difference of squares} \\ &= (a - b)(a - b)(a + b) \\ &= (a + b)(a - b)^2 \end{aligned}$$

5. Factor $3^8 - 1$: $(3^2 - 1)(3^2 + 1)(3^4 + 1)$. Since $3^2 - 1 = 9 - 1 = 8$, we know that 8 divides the product.

6. $5^{12} - 1 = (5^6 + 1)(5^3 + 1)(5^3 - 1)$. Calculate $5^3 + 1 = 125 + 1 = 126$, and $5^3 - 1 = 125 - 1 = 124$. So $126 + 124 = 250$

7. $a^3b - ab^3 = ab(a^2 - b^2) = ab(a + b)(a - b)$

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9. $(x + y)(x - y) + y^2 - 1 = x^2 - y^2 + y^2 - 1 = x^2 - 1 = (x + 1)(x - 1)$

10. Factor by grouping: $x^3 + 2x^2 - 16x - 32 = x^2(x + 2) - 16(x + 2) = (x + 2)(x^2 - 16) = (x + 2)(x + 4)(x - 4)$