1. $\boxed{5}$

2. $2^2 + 3^2 + 6^2 = 7^2 = \boxed{49}$

3. $\boxed{500}$

4. $\boxed{5}$

5. This problem is similar to Problem 9 on this page. We will apply the same idea. We will imagine the clocks as turning gears, with the minute hand as the moving point. At 6:00, the minute hand is pointing straight up. The first clock makes $\frac{59}{60}$ of a revolution in an hour, and the other clock makes $\frac{61}{60}$ of a revolution in an hour. We want to find some amount of hours, which I will call h, such that $\frac{59}{60}h$ and $\frac{61}{60}h$ are whole numbers. If the products are whole numbers, that means that in h hours, both clocks' minute hands have made whole turns and are back at the starting point. The obvious choice for h is 60. Then $\frac{59}{60}h = 59$ and $\frac{61}{60}h = 61$, so both clocks' minute hands have made a whole number of turns. The first clock's minute hand has made 59 turns and the second clock's minute hand has made 61 turns. After the end of 60 hours, the first clock is displaying 59 hours after 6:00, which is 5:00, and the second clock is displaying 61 hours after 6:00, which is 7:00.

   Now we know how long it takes for each clock to be displaying an hour again, which is after 60 hours. Now we need to find how long it will take for the two hours to be the same. After 60 hours, the first clock displays 5:00 and the second displays 7:00. After another 60 hours, the first clock displays 4:00 and the second displays 8:00. We continue around the clock until we see that it will be 6 rounds of 60 hours until the clocks both display 12:00. So $6 \times 60 = 360$ hours. The problem asks how many days it will be until they display the same time again, so we calculate $360 \div 24 = \boxed{15 \text{ days}}$.

6. Set up an equation such that the problem describes: let x be the first digit and y be the second digit, so we write $10x + y = x + y + xy$. Now we simplify by subtracting x and y from both sides: $9x = xy$. Now we divide both sides by x and get $9 = y$. So this tells us that the units digit of each of these numbers must be 9. So the 2 digit intergers with this property are $19, 29, 39, \ldots, 99$. The average of a sequence of numbers that are equally spaced is the middle term. So the middle term is $\boxed{59}$

7. We guess that Steve's grandma had to be born sometime between 1860's - 1920's. So let's think of two prime numbers that will multiply to such a number. Also, we want each number to be less than twice the other.

   Start with looking at the range of numbers by finding $\sqrt{1900} \approx 43$. So we want two numbers around 43. Now let's try some pairs in this range: $43^2 = 1849$. A little too low. $37 \times 53 = 1961$. Too high. $31 \times 61 = 1891$. Just right. Now we need to check that each numbers is less than twice the other. $31 < 2 \times 61$, check, and $61 < 2 \times 31$, check. So grandma was born in 1891, so her 90th birthday is on $\boxed{1981}$.

8. $\boxed{(1,3)}$

9. First let's change the rates to be per second. If one gear turns $33\frac{1}{3}$ times in a minute, then it turns $\frac{100}{3} \div 60 = \frac{5}{9}$ revs in a second. The second gear turns 45 times in a minute, so it

turns $\frac{3}{4}$ of a revolution in a second. We need to find some amount of seconds, which we will call x, such that $\frac{5}{9}x$ and $\frac{3}{4}x$ both produce a whole number. What number will cancel both denominators? $LCM(9,4) = 36$, so if $x = 36$, then $\frac{5}{9}x = 20$ and $\frac{3}{4}x = 27$. In other words, in 36 seconds, the first gear has made 20 rotations, and the second gear has made 27 rotations. But at the end of those 36 seconds, both gears are completing a full circle, and both are pointing north. $\boxed{36}$

10. Let's start with the idea that the people can make a perfect formation with less than 100 people. Guess and check.

    If he has 99 band members, he can make the following rectangles: 1×99, 3×33, 9×11, and their reverses. Let's try 3 and 33. If there are 33 rows with 3 members per row, then when we reverse the operations and make each row 2 people and 35 rows (with 2 people left over), that makes only 74 band members. When we try 3 rows with 33 people per row, we get 34 people in 1 row, which is 36 band members. Now let's try 9 and 11. If there are 9 people and 11 rows, then $8 \times 12 + 2 \neq 99$. If there are 11 people and 9 rows, then $10 \times 11 + 2 \neq 99$. We can conclude the amount is not 99 band members.

    Now let's try 98 band members. We only need to find one rectangular formation such that the movements work. Let's try 7 and 14. If there are 7 people in 14 rows, then when we reduce the number of people by 1 and increase the number of rows by 2, we get 6 people in 16 rows, which is 96 people, plus the 2 people left over. So we have found a formation that works when there are $\boxed{98}$ people.