Avid Academy
for Gifted Youth

Math Olympiad and Problem Solving Programs

E220 - Intermediate Math Competitions

Problem Set 15.1 - The Pascal Triangle

Name: Date:

1. $\boxed{x^4 + 4x^3 + 6x^2 + 4x + 1}$

2. $\boxed{x^6 - 3x^4y^2 + 3x^2y^4 - y^6}$

3. There was an error in the problem. The end term should have been 3^3, not just 3. If the end term was 3^3, then the answer would have been $(17+3)^3 = 20^3 = \boxed{8000}$, which I accepted. The true answer of the problem that was written was $\boxed{7976}$.

4. $\boxed{19487171}$

5. Two possibitilities. $\boxed{1,6,21,56,126}$ $\boxed{1,4,10,20,35,56,84}$

6. $2^10 = \boxed{1024}$

7. $\binom{8}{2} = \boxed{28}$

8. Free point.

9. This counting problem can be visualized as placing 3 heads into 7 available spots. We know we toss a coin 7 times, and we want combinations of 3 heads and 4 tails. So we think of it as placing our 3 heads in our sequence of 7, and then the extra blank 4 spots will be filled with tails. The way we do this is with combinatorics. So $\binom{7}{3} = \binom{7}{4} = \boxed{35}$

10. We need to find how many of $\binom{25}{n}$ will result in an odd number. We only need to do values of $0 \leq n \leq 13$ because they are symmetrical for the larger numbers for n. So if we find how many n's produce an odd number between 0 and 13, we will just multiply by 2 to count the n's between 14 and 25. Obviously, $\binom{25}{0} = 1$, so there is one odd. Then $\binom{25}{1} = 25$, which is odd. Now consider $\binom{25}{8} = 1081575$ and $\binom{25}{9} = 2042975$, so we have 2 more odds. So there are 4 odds between 0 and 13, so there are $2 \times 4 = \boxed{8}$ odd numbers in the 25th row.