

1. $\boxed{1,000,000}$
2. We know $a + bx = 15$ when $x = 2$ and $a + bx = 3$ when $x = 5$. So we know $a + 2b = 15$ and $a + 5b = 3$. Subtract the first equation from the second: $(a + 5b) - (a + 2b) = 3 - 15 \Rightarrow a + 5b - a - 2b = -12 \Rightarrow 3b = -12 \Rightarrow b = -4$. Then plugging this into the first equation: $a - 8 = 15 \Rightarrow a = 23$. So then $a + b = 23 - 4 = \boxed{19}$
3. To find its x and y intercepts, plug in 0 for x or y and solve. $(0) - 4 = 4(x - 8) \Rightarrow -1 = x - 8 \Rightarrow x = 7$. Now do $y - 4 = 4((0) - 8) \Rightarrow y - 4 = -32 \Rightarrow y = -28$. So the sum of the intercepts is $-28 + 7 = \boxed{-21}$
4. First let's find where $y = 5x + 3$ and $y = -2x - 25$ intersect by setting them equal to each other: $5x + 3 = -2x - 25 \Rightarrow 7x = -28 \Rightarrow x = -4$, and so $y = 5(-4) + 3 = -17$. So if $y = 3x + k$ intersects these lines at $(-4, -17)$, then we plug in: $-17 = 3(-4) + k \Rightarrow k = \boxed{-5}$
5. $\boxed{3}$
6. Remember the quadratic equation: for an equation of the form $ax^2 + bx + c$, the solution is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
So if x_1, x_2 are the solutions to the equation, then $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$. So $(x_1 - 1)(x_2 - 1) =$

$$\begin{aligned} & \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} - 1 \right) \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} - 1 \right) \\ &= \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} - \frac{2a}{2a} \right) \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} - \frac{2a}{2a} \right) \\ &= \left(\frac{(-b - 2a) + \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{(-b - 2a) - \sqrt{b^2 - 4ac}}{2a} \right) \text{ (notice difference of squares!)} \\ &= \frac{(-b - 2a)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2} \\ &= \frac{b^2 + 4ab + 4a^2 - b^2 - 4ac}{4a^2} \\ &= \frac{4a(b + a + c)}{4a^2} = \frac{a + b + c}{a} \end{aligned}$$

So in our quadratic formula, we have $a = 1$, $b = -5$, and $c = 9$, so then $\frac{a + b + c}{a} = \frac{1 - 5 + 9}{1} = \boxed{5}$



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7. Classic problem. First, square both sides: $(x + \frac{1}{x})^2 = 6^2 \Rightarrow x^2 + 2(x)(\frac{1}{x}) + (\frac{1}{x})^2 = 36 \Rightarrow x^2 + 2 + \frac{1}{x^2} = 36 \Rightarrow x^2 + \frac{1}{x^2} = \boxed{34}$
8.
9.
10.