

1.  $\boxed{-11}$

2.  $\boxed{-4}$

3. You can never ever ever just drop absolute values. You have to make conditions first.

Case 1: the things inside the absolute values are positive. Then  $x - 1 \geq 0$ , or  $x \geq 1$ . If we know that  $x \geq 1$ , then  $x + 1 \geq 0$  and  $x - 1 \geq 0$ , so now we can drop the absolute values. Then we reduce to  $x + 1 - x + 1 = 2$ . So  $\boxed{\text{if } x \geq 1, \text{ we simplify to } 2}$ .

Case 2: the thing inside one abs. value is positive and the other is negative. This happens when  $x - 1 < 0$  and  $x + 1 > 0$ , or when  $x < 1$  and  $x > -1$ . So while  $-1 < x < 1$ , then  $x - 1$  is negative, so in order to drop the abs. value signs, we have to put a negative sign in front to cancel the negative and make it positive. So  $|x + 1| - |x - 1|$  becomes  $x + 1 - (-(x - 1)) = x + 1 + (x - 1) = 2x$ . So  $\boxed{\text{if } -1 < x < 1, \text{ we simplify to } 2x}$ .

Case 3: both things inside the abs. values are negative. Then  $x + 1 \leq 0$ , or  $x \leq -1$ . Now we have to put negatives in front of both quantities, and we get  $-(x + 1) - (-(x - 1)) = -x - 1 + x - 1 = -2$ . So  $\boxed{\text{if } x \leq -1, \text{ we simplify to } -2}$ .

A correct answer has all three parts.

4. Use the same method as the previous problem.

Case 1: the things inside the absolute values are positive. Then  $x - \frac{1}{3} \geq 0$ , or  $x \geq \frac{1}{3}$ . If we know that  $x \geq \frac{1}{3}$ , then  $x + \frac{1}{3} \geq 0$  and  $x - \frac{1}{3} \geq 0$ , so now we can drop the absolute values. Then we reduce to  $x - \frac{1}{3} + x + \frac{1}{3} = 2x$ . So  $\boxed{\text{if } x \geq \frac{1}{3}, \text{ we simplify to } 2x}$ .

Case 2: the thing inside one abs. value is positive and the other is negative. This happens when  $x - \frac{1}{3} < 0$  and  $x + \frac{1}{3} > 0$ , or when  $x < \frac{1}{3}$  and  $x > -\frac{1}{3}$ . So while  $-\frac{1}{3} < x < \frac{1}{3}$ , then  $x - \frac{1}{3}$  is negative, so in order to drop the abs. value signs, we have to put a negative sign in front to cancel the negative and make it positive. So  $|x - \frac{1}{3}| + |x + \frac{1}{3}|$  becomes  $-(x - \frac{1}{3}) + x + \frac{1}{3} = -x + \frac{1}{3} + x + \frac{1}{3} = \frac{2}{3}$ . So  $\boxed{\text{if } -\frac{1}{3} < x < \frac{1}{3}, \text{ we simplify to } \frac{2}{3}}$ .

Case 3: both things inside the abs. values are negative. Then  $x + \frac{1}{3} \leq 0$ , or  $x \leq -\frac{1}{3}$ . Now we have to put negatives in front of both quantities, and we get  $-(x - \frac{1}{3}) - (x + \frac{1}{3}) = -x + \frac{1}{3} - x - \frac{1}{3} = -2x$ . So  $\boxed{\text{if } x \leq -\frac{1}{3}, \text{ we simplify to } -2x}$ .

A correct answer has all three parts.

5. If  $|x| = 5\frac{2}{3}$ ,  $|y| = 1\frac{1}{3}$ , then  $x = \pm 5\frac{2}{3}$  and  $y = \pm 1\frac{1}{3}$ . So the expression  $x - y$  could have four possibilities:

$$5\frac{2}{3} + 1\frac{1}{3} = 7$$

$$5\frac{2}{3} - 1\frac{1}{3} = 4\frac{1}{3}$$

$$-5\frac{2}{3} + 1\frac{1}{3} = -4\frac{1}{3}$$

$$-5\frac{2}{3} - 1\frac{1}{3} = -7$$

So  $x - y = \boxed{\pm 7, \pm 4\frac{1}{3}}$



Math Olympiad and Problem Solving Programs  
E220 - Intermediate Math Competitions  
Problem Set 14.1 - Absolute Values

Name:

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6. Again, there are 3 cases.

Case 1:  $|a| < |b| < |c|$ . Imagine that  $c = 10$ ,  $b = -5$ , and  $a = -1$  if it helps you visualize it. Then  $a + c > 0$ ,  $b + c > 0$ , and  $a + b < 0$ . So we put a negative around  $a + b$  and reduce:  $|a + c| + |b + c| - |a + b| = a + c + b + c - (-(a + b)) = a + c + b + c + a + b = \boxed{2(a+b+c)}$

Case 2:  $|a| < |c| < |b|$ . Imagine that  $b = -10$ ,  $c = 5$ , and  $a = -1$  if it helps you visualize it. Then  $a + c > 0$ ,  $b + c < 0$ , and  $a + b < 0$ . So we put a negative around  $a + b$  and  $b + c$  and reduce:  $|a + c| + |b + c| - |a + b| = a + c - (b + c) - (-(a + b)) = a + c - b - c + a + b = \boxed{2a}$

Case 2:  $|c| < |a| < |b|$ . Imagine that  $b = -10$ ,  $a = -5$ , and  $c = 1$  if it helps you visualize it. Then  $a + c < 0$ ,  $b + c < 0$ , and  $a + b < 0$ . So we put a negative around each quantity and reduce:  $|a + c| + |b + c| - |a + b| = -(a + c) - (b + c) - (-(a + b)) = -a - c - b - c + a + b = \boxed{-2c}$

A correct answer has all three parts.

7.  $\boxed{-15}$   
8.  $\boxed{1}$   
9.  $\boxed{8}$   
10.  $\boxed{2}$