



Math Olympiad and Problem Solving Programs  
E220 - Intermediate Math Competitions  
Problem Set 10.2 - Counting and Probability

Name:

Date:

1.  $\boxed{18}$
2. First do some calculations (the brackets mean to round down to the nearest whole number).  
 $\lfloor 999 \div 2 \rfloor = 499$ .  $\lfloor 99 \div 2 \rfloor = 49$ .  
 $\lfloor 999 \div 3 \rfloor = 333$ .  $\lfloor 99 \div 3 \rfloor = 33$ .  
 $\lfloor 999 \div 6 \rfloor = 166$ .  $\lfloor 99 \div 6 \rfloor = 16$ .  
So there are  $499 - 49 = 450$  3-digit numbers that are divisible by 2. There are  $333 - 33 = 300$  numbers that are divisible by 3. But we have overcounted the numbers divisible by 6; there are  $166 - 16 = 150$  numbers divisible by 6. So there are  $450 + 300 - 150 = \boxed{600}$  3-digit numbers that are divisible by 2 or 3.
3. An even three-digit numbers that is divisible by 9 will be divisible by 18. So  $\lfloor 999 \div 18 \rfloor = 55$ , and  $\lfloor 99 \div 18 \rfloor = 5$ . So there are  $55 - 5 = \boxed{50}$  even 3-digit numbers divisible by 9.
4. When we want to know how many words we can create with the letters of the word PROBABILITY, we get  $\frac{11!}{2!2!}$ , or the number of digits divided by the number of repeat letters. But we only want 4-letter words. So think of this problem as creating 11-letter words, chopping off the last 7 letters, and taking the first four letters as your word. But there is overcounting with that as well. For every combination of the first four letters, there are  $7!$  ways to arrange the last 7 letters, so there are  $7!$  repeats we have to divide out. So the answer is  $\frac{11!}{2!2!} \div 7! = \frac{11!}{7!2!2!} = \boxed{1980}$
5.  $\binom{14}{4} = \boxed{1001}$
6.  $\binom{5}{2} \binom{4}{2} \binom{3}{2} = \boxed{180}$
7.  $\boxed{15}$
8. There are  $2^4 = 16$  possible combinations of 4 coin flips. There are 4 ways that 3 heads appear: HHHT, HHTH, HTHH, THHH. There is one way 4 heads can appear: HHHH. So the probability that at least 3 heads appear =  $\text{prob}(3 \text{ heads appear}) + \text{prob}(1 \text{ head appears})$   
 $= \frac{4}{16} + \frac{1}{16} = \boxed{\frac{5}{16}}$
9. I product divisible by 3 must have three in it. So count the pairings of numbers that have at least one 3 in it: (1,3), (2,3), (3,3), (4,3), (3,1), (3,2). So of the  $4 \times 3 = 12$  combinations of numbers we can get, 6 of the will be divisible by 3. So  $\frac{6}{12} = \boxed{\frac{1}{2}}$
10. Let's take the two probabilities of each event and multiply them together.  
What is the probability of getting a card that is a color? 1, because no matter what, your card will be either red or black.  
What's the probability that the second card is not the same color as the first one? There are 51 cards left to choose from. Half of the 52 cards are red, half are black, so there are 26 cards of the color opposite the one we already have. So the probability of drawing a card of opposite color is  $\frac{26}{51}$ .



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Now we multiply the probabilities from the two steps.  $1 \times \frac{26}{51} = \boxed{\frac{26}{51}}$