1. $2010 = 2 \cdot 3 \cdot 5 \cdot 67$. So there are $(1+1)(1+1)(1+1)(1+1) = 2^4 = \boxed{16}$
2. Factor out the square components: $N = 2^3 \cdot 3^4 \cdot 5^3 = (2 \times 3^2 \times 5)^2 \times 2 \times 5$. Now do the factor counting on the squared term: $(1+1)(2+1)(1+1) = 2 \times 3 \times 2 = \boxed{12}$
3. Let $n = 2^4 \cdot 3^5 \cdot 4^6 \cdot 6^7 = 2^4 \cdot 3^5 \cdot (2^2)^6 \cdot (2^7 \cdot 3^7) = 2^{23} \cdot 12$. There are $\boxed{312}$ factors.
4. $2^2 \times 3^2 \times 5 = (2^2 \times 3) \times 3 \times 5$. There are $\boxed{4}$ factors.
5. The number $n = 3^4 \cdot 5^3 \cdot 7$ does not have any 2's, so n can never be even. $\boxed{0}$
6. If n has exactly three natural-number factors, then n is a square number of a prime number, like 25. So n^4 has $\boxed{9}$ factors.
7. $\boxed{80}$
8. $\boxed{24}$
9. $\boxed{30}$
10. The number with the greatest number of factors is 12, because it factors to $2^2 \times 3$, so it has 6 factors. So make $b = 12$, and make n the greatest possible number, so $n = 15$. So $(2^2 \times 3)^1 5 = 2^{30} \times 3^{15}$, and it has 21×16 factors. $\boxed{496}$