



Math Olympiad and Problem Solving Programs  
E210 - Introductory Math Competitions  
Problem Set 28.1 - Probability

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1.  $P(\text{yellow}) = \frac{1}{2}$ ,  $P(\text{blue}) = \frac{1}{3}$ ,  $P(\text{green}) = \frac{1}{6}$ .

(a)  $P(\text{green, blue}) = \frac{1}{6} \cdot \frac{1}{3} = \boxed{\frac{1}{18}}$

(b)  $P(\text{yellow, green}) = \frac{1}{2} \cdot \frac{1}{6} = \boxed{\frac{1}{12}}$

(c)  $P(\text{not green, blue}) = (1 - \frac{1}{6}) \cdot \frac{1}{3} = \frac{5}{6} \cdot \frac{1}{3} = \boxed{\frac{5}{18}}$

(d)  $P(\text{blue, blue}) = \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{1}{9}}$

(e)  $P(\text{not blue, not green}) = (1 - \frac{1}{3}) \cdot (1 - \frac{1}{6}) = \frac{2}{3} \cdot \frac{5}{6} = \boxed{\frac{5}{9}}$

2.  $P(\text{tail}) = \frac{1}{2}$ ,  $P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$ . Then  $\frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}}$

3.  $\frac{1}{6} + \frac{1}{6} = \boxed{\frac{1}{3}}$

4. Since the three choices have equal probability,  $P(\text{top}) = \frac{1}{3}$ ,  $P(\text{bottom}) = \frac{1}{3}$ , and  $P(\text{side}) = \frac{1}{3}$ . The problem asks for AT LEAST one top down, which means we can get 1 top down, 2 top down, or 3 top down. Rather than calculating these three probabilities and adding, let's make it easier by calculating the probability we get NO top downs, and subtracting that from 1.

The probability of getting not a top down is  $1 - \frac{1}{3} = \frac{2}{3}$ . The probability of getting three not-top-downs in a row is  $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$ . So the probability of getting at least one top down is  $1 - \frac{8}{27} = \boxed{\frac{19}{27}}$

5.  $\boxed{\frac{1}{8}}$

6. Let's write the possible combinations of 4 coin tosses:

all heads: *HHHH*

3 heads, 1 tail: *HHHT HHTH HTHH THHH*

2 heads, 2 tails: *HHTT TTHH HTHT THTH HTTH THHT*

1 head, 3 tails: *TTTH TTHT THTT HTTT*

all tails: *TTTT*

We can see there are  $2^4 = 16$  different combinations. We count there are 11 ways of getting at least 2 heads (meaning 2 heads, 3 heads, or 4 heads), so the probability of getting at least 2 heads is  $\boxed{\frac{11}{16}}$ .

7.  $\boxed{\frac{3}{10}}$

8. How can the product of two numbers be prime? One has to be 1, and the other has to be prime. Make a probability table, and only complete the squares where this combination occurs.



Math Olympiad and Problem Solving Programs  
E210 - Introductory Math Competitions  
Problem Set 28.1 - Probability

Name: \_\_\_\_\_

Date: \_\_\_\_\_

	1	2	3	4	5	6
1		2	3		5	
2	2					
3	3					
4						
5	5					
6						

We can see that of the 36 combinations of products, only 6 of them have prime products. So the probability is  $\frac{6}{36} = \boxed{\frac{1}{6}}$

9. Since we are drawing without replacement, we have to change the probabilities each time.

The letters of the word ACADEMY are written on seven cards with one letter per card. The cards were thoroughly shuffled and spread out face down in front of you. Think about drawing cards without replacement. Find the following probabilities (note Y is not a vowel).

(a)  $P(A, A) = \frac{2}{7} \cdot \frac{1}{6} = \boxed{\frac{1}{21}}$

(b)  $P(\text{not } A, A) = (1 - \frac{2}{7}) \cdot \frac{2}{6} = \frac{5}{7} \cdot \frac{1}{3} = \boxed{\frac{5}{21}}$

(c)  $P(C, D, A) = \frac{1}{7} \cdot \frac{1}{6} \cdot \frac{2}{5} = \boxed{\frac{1}{105}}$

(d)  $P(\text{not } A, \text{not } A, A, A) = \frac{5}{7} \cdot \frac{4}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} = \boxed{\frac{1}{21}}$

(e)  $P(\text{vowel, vowel, M}) = \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} = \boxed{\frac{1}{35}}$

10. A number is a multiple of 3 if the sum of the digits is divisible by 3.

First, let's see how many different numbers we can make with these digits. We can make  $5 \times 4 = 20$  different numbers. Now let's see how many of these combinations have sums divisible by 3:

13	no	15	yes	17	no	19	no
35	no	37	no	39	yes		
57	yes	59	no				
79	no						

So 15, 51, 39, 93, 57, and 75 are multiples of 3. So of the 20 possible numbers, 6 are multiples.

The probability then is  $\frac{6}{20} = \boxed{\frac{3}{10}}$