Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 27.2 - Modulo Residues
Name: Date:

Instruction: Calculate the following modulo RESIDUES. That means no negative answers or answers that are greater than the mod.

1. $35 + 17 \pmod 5 \equiv 0 + 2 \equiv \boxed{2}$
2. $11 \cdot 13 \pmod 6 \equiv 5 \cdot 1 \equiv \boxed{5}$
3. $11 - 13 = -2 \pmod 6 \equiv \boxed{4}$
4. $13 - 10 = 3 \pmod 3 \equiv \boxed{0}$
5. $12 \cdot 10 \pmod 5 \equiv 2 \cdot 0 \equiv \boxed{0}$
6. $39 + 11 \pmod 5 \equiv 4 + 1 \equiv \boxed{0}$
7. $10 \cdot 10 \pmod{11} \equiv -1 \cdot -1 \equiv \boxed{1}$
8. $12 \cdot 10 \pmod 3 \equiv 0 \cdot 1 \equiv \boxed{0}$
9. $13 \cdot 5 \pmod 6 \equiv 1 \cdot -1 \equiv -1 \equiv \boxed{5}$
10. $5^4 = 625 \pmod{11} \equiv \boxed{9}$
11. $21 \cdot 17 + 7 \cdot 17 \pmod 7 \equiv 0 \cdot 3 + 0 \cdot 3 \equiv 0 + 0 \equiv \boxed{0}$
12. Notice that $2^3 = 8 \equiv 1 \pmod 7$. Since $2^3 \equiv 1 \pmod 7$, if we square both sides, we get $(2^3)^2 \equiv 1^2 \pmod 7$, which is $2^6 \equiv 1 \pmod 7$. So $2^6 \pmod 7 \equiv \boxed{1}$.
13. Notice that $3^2 = 9 \equiv -1 \pmod{10}$. Since $3^2 \equiv -1 \pmod{10}$, if we raise both sides to the 30th power, we have $(3^2)^{30} \equiv (-1)^{30} \pmod{10}$, which is $3^{60} \equiv 1 \pmod{10}$. So $3^{60} \pmod{10} \equiv \boxed{1}$.
14. Notice $2^2 = 4 \equiv -1 \pmod 5$. Since $2^2 \equiv -1 \pmod 5$, if we raise both sides to the 30th power, we have $(2^2)^{30} \equiv (-1)^{30} \pmod 5$, which is $2^{60} \equiv 1 \pmod 5$. We are almost done, we just need to multiply both sides by 2^2, so we have $2^{60} \cdot 2^2 \equiv 1 \cdot 2^2 \pmod 5$. $2^{60} \cdot 2^2 = 2^{62}$, which is what we are looking for. So $2^{62} \equiv 2^2 \pmod 5$, or $2^{62} \pmod 5 \equiv \boxed{4}$.