



Math Olympiad and Problem Solving Programs  
F130 - Advanced Problem Solving  
Problem Set 27.1 - Modular Operations

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. In the table, we take *left side number* and subtract *top side number*. For the first entry, we have  $0 - 0 = 0$ . For the next entry to the right of 0, we have  $0 - 1 = -1$ , which is  $-1 + 7 = 6 \pmod{7}$ . Continue to create the table below.

Modulo 7 Subtraction

$\ominus$	0	1	2	3	4	5	6
0	0	6	5	4	3	2	1
1	1	0	6	5	4	3	2
2	2	1	0	6	5	4	3
3	3	2	1	0	6	5	4
4	4	3	2	1	0	6	5
5	5	4	3	2	1	0	6
6	6	5	4	3	2	1	0

2. (a)  True (b)  False
3.  2
4.  $2001 \times 2003 \times 2005 \times 2007 \times 2009 \times 2011 \pmod{9}$   
 $= 3 \times 5 \times 7 \times 0 \times 2 \times 4 = \boxed{0}$
5. (a)  2 (b)  0 (c)  0
6.  0
7. Numbers that are congruent to 5 (mod 7) are of the form  $7k + 5$ . We want all the numbers  $k$  such that  $1000 \leq 7k + 5 \leq 2000$ . Let's solve this inequality by first subtracting all sides by 5:  $995 \leq 7k \leq 1995$ . Now we divide all sides by 7:  $142.143 \leq k \leq 285$ . So  $k$  can be any of the numbers in the list 143, 144, 145, ..., 284, 285. How do we count the numbers in this list? Subtract the smallest from the largest and add one:  $285 - 143 + 1 = \boxed{143}$
8. (a) It is probably easiest to just compute the sum, which is 3, and find what 3 is mod 6, which is just  3.
- Or we can find the mod 6 of each number.  $12 \pmod{6} \equiv 0$ ,  $11 \pmod{6} \equiv 5$ ,  $10 \pmod{6} \equiv 4$ ,  $9 \pmod{6} \equiv 3$ ,  $8 \pmod{6} \equiv 2$ , and  $7 \pmod{6} \equiv 1$ . Now we have  $12 - 11 + 10 - 9 + 8 - 7 \equiv 0 - 5 + 4 - 3 + 2 - 1 \pmod{6} \equiv -5 + 1 + 1 \equiv -3 \pmod{6} \equiv \boxed{3}$
- (b) Let's find what  $101 \pmod{6}$  is. We can find that  $101 \equiv 5 \pmod{6} \equiv -1 \pmod{6}$ . Now we take the equivalence  $101 \equiv -1 \pmod{6}$  and raise both numbers to the 99th power:  $(101)^{99} \equiv (-1)^{99} \pmod{6}$ . What is  $-1^{99}$ ? It is  $-1 \times -1 \times -1 \times \dots \times -1$  99 times, which is  $-1$ . So  $(101)^{99} \equiv (-1)^{99} \pmod{6} \equiv -1 \pmod{6}$ . Since  $-1 \equiv 5 \pmod{6}$ , then the residue of  $101^{99} \pmod{6}$  is  5
9. Altogether they have  $83 + 129 = 212$  quarters and  $159 + 266 = 425$  dimes. Now we need to find what coins will be left over after they put them in rolls, and we will use mods.  $212 \equiv 12 \pmod{40}$  (we use mod 40 because the quarter rolls contain 40 quarters), and  $425 \equiv 25 \pmod{50}$ . So there are 12 quarters left and 25 dimes. The quarters are worth  $12 \times 0.25 = \$3.00$ , and the dimes are worth  $25 \times 0.10 = \$2.50$ , so the total worth of the leftover coins is  \$5.50.



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10. If each bag contains a quarter, a dime, a nickel, and a penny, then each bag contains  $0.25 + 0.10 + 0.05 + 0.01 = \$0.41$ . She has 73 of these, so she has a total of  $73 \times \$0.41 = \$29.93$ . If she exchanges this amount for dollars, dimes, and pennies, then she will get 29 dollars, 9 dimes, and 3 pennies.