Math Olympiad and Problem Solving Programs
F130 - Advanced Problem Solving
Problem Set 25.1 - Binary Arithmetic

Avid Academy for Gifted Youth

Name: Date:

1. (a) $\boxed{101000_2}$ (b) $\boxed{111000_2}$ (c) $\boxed{11001_2}$
2. (a) $\boxed{1010_2}$ (b) $\boxed{101_2}$ (c) $\boxed{1101_2}$
3. (a) $\boxed{x = 10001_2}$ (b) $\boxed{x = 11000_2}$ (c) $\boxed{x = 110_2}$
4. (a) $\boxed{1111011_2}$ (b) $\boxed{11011_2}$ (c) $\boxed{1010010001_2}$ (d) $\boxed{1001_2}$
5. (a) $\boxed{1000010_2}$ (b) $\boxed{11001_2}$

   (c) First do $10011_2 - 10001_2 = 10_2$. Now add $1101_2 + 10_2 = \boxed{1111_2}$
6. (a) $\boxed{101011111000_2}$ (b) $\boxed{1011_2}$
7. We can see that 10011001_2 is an odd number, because it's last digit is 1, so it will be *some even numbers* + 1 when we convert to base 10. We know 10_2 is 2 in base 10, so when we divide an odd number by 2, there will be remainder $\boxed{1}$.
8. (a) $\boxed{11001_2 = 25_{10}}$ (b) $\boxed{1010001_2 = 81_{10}}$
9. Difference of squares formula is very important! Memorize this! $a^2 - b^2 = (a+b)(a-b)$.

   Now we see that $(111111_2)^2 - (111110_2)^2$ is *squared thing* − *squared thing*, so we can use the difference of squares formula. We have $(111111_2)^2 - (111110_2)^2 = (111111_2 + 111110_2)(111111_2 - 111110_2)$. The addition becomes 1111101_2, and the subtraction is 1_2. So the multiplication of $1111101_2 \times 1_2 = \boxed{1111101_2}$
10. Write $2^{16} - 1$ in binary: $10000_2 - 1_2 = 1111_2$. Now convert 7 to binary: 111_2. Now we divide, and we get that 111_2 goes into 1111_2 10 times with remainder 1. $\boxed{\text{no, R1}}$