



Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 22.2 - Rational and Irrational Numbers

Name:

Date:

1. B

2. D

3. $\frac{7}{10}, \frac{3}{4}, \frac{4}{5}$

4. $\frac{5}{9}, \frac{7}{12}, \frac{3}{4}$

5. $\frac{33}{34}, \frac{22}{23}, \frac{11}{12}$

6. C

7. Recall: a rational number is a whole number or a number that can be represented as a fraction. Any repeating decimal can be written as a fraction.

(A) $4.\overline{6} = 4\frac{6}{9} = 4\frac{2}{3}$. This number can be represented as a fraction, so it is rational.

(B) $\frac{13}{21}$. This number is a fraction, so it is rational.

(C) $\sqrt{9} = 3$, which is clearly rational.

(D) $\sqrt{18} = 3\sqrt{2}$. This number can never be written as a fraction, so it is irrational.

8. When writing repeating decimals, we take as many digits as are repeated and place them in the numerator, then we put as many 9's in the denominator.

(A) $0.\overline{4} = \frac{4}{9}$

(B) $3.\overline{3} = 3\frac{3}{9} = 3\frac{1}{3} = \frac{10}{3}$

(C) $0.\overline{45} = \frac{45}{99} = \frac{5}{11}$

(D) $0.\overline{9} = \frac{9}{9} = \frac{1}{1}$

9. Strategy. Let x equal the decimal. Multiply both sides by 10^n where n is the number of repeating digits. Then subtract the two equations and solve for x .

(A)

$$x = 0.583333 \dots$$

$$10x = 5.833333 \dots$$

Subtract the top from the bottom:

$$10x - x = 5.833333 \dots - 0.583333 \dots$$

$$9x = 5.83 - 0.58 = 5.25 = \frac{525}{100} = \frac{21}{4}$$

$$x = \frac{21}{4} \cdot \frac{1}{9} = \frac{21}{36} = \frac{7}{12}$$



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(B) $\boxed{\frac{1}{3}}$

(C)

$$\begin{aligned}x &= 0.67777\dots \\10x &= 6.77777\dots\end{aligned}$$

Subtract the top from the bottom:

$$\begin{aligned}10x - x &= 6.77777\dots - 0.67777\dots \\9x &= 6.7 - 0.6 = 6.1 = \frac{61}{10} \\x &= \frac{61}{10} \cdot \frac{1}{9} = \boxed{\frac{61}{90}}\end{aligned}$$

(D)

$$\begin{aligned}x &= 0.29545454\dots \\100x &= 29.54545454\dots\end{aligned}$$

Subtract the top from the bottom:

$$\begin{aligned}100x - x &= 29.54545454\dots - 0.29545454\dots \\99x &= 29.54 - .29 = 29.25 = \frac{2925}{100} = \frac{117}{4} \\x &= \frac{117}{4} \cdot \frac{1}{99} = \frac{117}{396} = \boxed{\frac{13}{44}}\end{aligned}$$

10. Which of the following statements are true?

I. The sum of two rational numbers are rational numbers: true

Proof: A rational number can be expressed as a fraction. Let two rational numbers be $\frac{a}{b}$ and $\frac{c}{d}$. Then when we add them, we get $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{cb}{bd} = \frac{ad+cd}{bd}$. Now we have some number that can be represented as a fraction, so the sum of two rational numbers is rational.

II. The sum of two irrational numbers are irrational numbers: true.

III. The sum of one rational number and one irrational number is a rational number: false.

Proof by counter example: 1 is rational, and $\sqrt{2}$ is irrational. Their sum, $1 + \sqrt{2}$, is still irrational.

\boxed{C}