

1. (a) $\boxed{40,320}$ (b) $\boxed{1320}$ (c) $\boxed{420}$

2. (a) $6! \cdot 56 = 6! \cdot 7 \cdot 8 = \boxed{8!}$ (b) $120 \cdot \frac{10!}{5!} = 5! \cdot \frac{10!}{5!} = \boxed{10!}$ (c) $\boxed{n(n-1)(n-2)}$

3. $6 \times 4 = \boxed{24}$

4. $\underline{5} \cdot \underline{5} \cdot \underline{5} \cdot \underline{5} = 5^4 = \boxed{625}$

5. $4! = \boxed{24}$

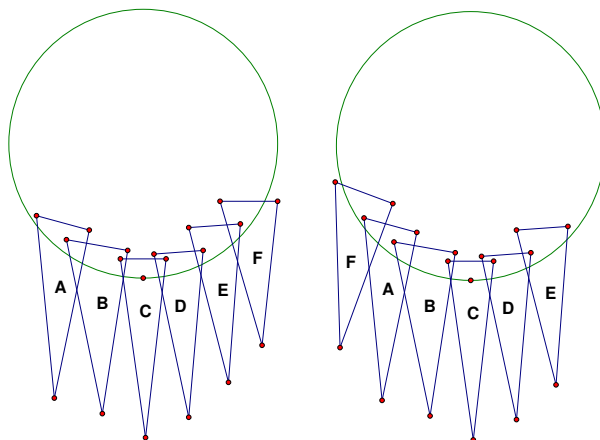
6. There are 7 digits, each of which can be one of the letters of the alphabet. The problem does NOT say that we can't repeat letters, so we assume that we can repeat letters.

$$\underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} = \boxed{26^7}$$

7. 9 small triangles + 3 medium triangles + 1 big triangle = $\boxed{13}$

8. We only get to pick a soup OR a salad, and there are 3 salads and 4 soups, so there are 7 ways to pick this appetizer. So there are $7 \times 5 \times 6 = \boxed{210}$ ways to pick a meal.

9. Our natural thought when we see this problem is to say $6!$, because Dr. Li is arranging 6 objects. However, this is overcounting because the objects are arranged in a circle. In order to understand this idea, let's draw a picture of a key chain. If you look at my diagram below (the triangles are keys), I have labeled the keys A-F. They are currently arranged in alphabetical order. However, if we flip key F around the chain, they are still the in same order; all the same keys are still next to each other.



If you laid the key ring on a table and spaced out all the keys evenly, it would still be the same arrangement. But you can flip they keys 6 times. So for every arrangement, there are 6 ways of overcounting. So we have to divide $6!$ by the overcounts, or by 6. So the answer is $6! \div 6 = \boxed{5! = 120}$

10. Four boys and three girls line up to take a group picture. How many ways are there to arrange the children in each of the following situations?



Math Olympiad and Problem Solving Programs
 E210 - Introductory Math Competitions
 Problem Set 21.1 - Permutations

Name:

Date:

(a) $7! = 5040$

(b)

$$\text{Back} : \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4}$$

$$\text{Front} : \underline{3} \cdot \underline{2} \cdot \underline{1}$$

$7! = 5040$

(c)

$$\text{Back(boys)} : \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$$

$$\text{Front(girls)} : \underline{3} \cdot \underline{2} \cdot \underline{1}$$

$3!4! = 144$

(d)

$$\underline{B} \cdot \underline{B} \cdot [\underline{G} \cdot \underline{G} \cdot \underline{G}] \cdot \underline{B} \cdot \underline{B}$$

$$\underline{4} \cdot \underline{3} \cdot [\underline{3} \cdot \underline{2} \cdot \underline{1}] \cdot \underline{2} \cdot \underline{1}$$

$3!4! = 144$

(e) The easiest way to do this problem is to count the ways the students line up with no girls next to each other. We must organize our counting to make sure we don't make any mistakes.

$$\underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{b}$$

$$\underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b}$$

$$\underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G}$$

$$\underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b}$$

$$\underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G}$$

$$\underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G}$$

$$\underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b}$$

$$\underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G}$$

$$\underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G}$$

$$\underline{b} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G} \cdot \underline{b} \cdot \underline{G}$$

So there are 10 ways to arrange the students so that no girls are together. In each of the arrangements, there are $3!4!$ ways of arranging the students, like we found in part d. So there are $10 \cdot 3!4! = 1440$ ways to line up the students this way.

(f) We already found how many ways we can arrange the students with no girls together in part e, which is 1440. We also found how many total ways to arrange the students in part a, which is 5040. So all we have to do now is subtract. $5040 - 1440 = 3600$