

- (a)  $\boxed{2.25}$  (b)  $\boxed{0.0\overline{89}}$  (c)  $\boxed{-0.325}$
- A reciprocal is the fraction flip of a number. The first 10 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Their reciprocals are  $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9},$  and  $\frac{1}{10}$ . The decimal equivalents of these fractions are  $1, 0.5, 0.\overline{3}, 0.25, 0.2, 0.1\overline{6}, 0.142857, 0.125, 0.\overline{1},$  and  $0.1$ . So there are  $\boxed{4}$  repeating reciprocals.
- $\frac{6! + 5!}{5! + 4!} = \frac{720 + 120}{120 + 24} = \frac{840}{144} = \frac{35}{6} = \boxed{5.8\overline{3}}$
- $\boxed{7}$
- $\boxed{\frac{4}{9}}$
- To find repeating decimal's corresponding fraction, you find the number of repeating digits and divide by that many 9's like in part b and c. Part d requires algebra.
  - $\boxed{\frac{21}{5}}$
  - $7.\overline{6} = 7\frac{6}{9} = 7\frac{2}{3} = \boxed{\frac{23}{3}}$
  - $-3.\overline{428} = -3\frac{428}{999} = \boxed{-\frac{3425}{999}}$
  - Let  $x$  be the fraction that corresponds to  $0.1\overline{27}$ . So we have  $x = 0.1\overline{27}$ . Now multiply both sides by 100 (we choose 100 because there are 2 repeating digits, so we multiply by the multiple of 10 with 2 zeros). So we have  $100x = 12.7\overline{27}$ . Now we consider the equations side by side:
 
$$100x = 12.7272727 \dots$$

$$x = 0.1272727 \dots$$
 Now subtract the bottom equation from the top equation, and we get  $100x - x = 99x$  on the left, and  $12.72727\dots - 0.12727\dots = 12.7 - 0.1 = 12.6$ . So we have  $99x = 12.6 = 12\frac{6}{10} = 12\frac{3}{5} = \frac{63}{5}$ . Now we solve for  $x$  by dividing both sides by 99:  $x = \frac{63}{5} \times \frac{1}{99} = \frac{63}{495} = \boxed{\frac{7}{55}}$
- Write  $0.\overline{9}$  as a fraction:  $0.\overline{9} = \frac{9}{9} = 1$ . So there is  $\boxed{\text{no difference}}$  between the 1 and  $0.\overline{9}$ .
- $10^1 = 10$  has 1 zero,  $10^2 = 100$  has 2 zeros,  $10^3 = 1000$  has 3 zeros. Notice the pattern: 10 raised to the  $x$  power has  $x$  zeros. So  $10^{30}$  has 30 zeros. The total digits of the decimal expansion (or written out without exponents) is 30 zeros plus the 1 at the beginning. So  $\boxed{31}$  digits.
- We know  $2^{10} = 1024$ . So  $2^{30} = (2^{10})^3 = (1024)^3 \approx (1000)^3 = (10^3)^3 = 10^9$ . We know from the previous problem that this number has  $\boxed{10}$  digits.
- Notice  $10^{30} = 2^{30} \times 5^{30}$ . Since  $10^{30}$  has 31 digits and  $2^{30}$  has 10 digits,  $5^{30}$  has  $31 - 10 = \boxed{21}$  digits.