1. In order for a perfect square to be divisible by 8, or 2^3, it needs to be divisible by 16, or $2^4 = 4^2$. So we want to find how many numbers n satisfy the following inequality:

$$(4n)^2 < 1000$$

We solve by square-rooting both sides: $4n < \sqrt{1000}$ or $4n < 31$. Now divide by 4: $n < 7.9$. So the highest whole number less than 7.9 is $\boxed{7}$. The 7 perfect squares less than 1000 that are divisible by 8 are 16, 64, 144, 256, 400, 576, and 784.

2. $\boxed{45}$

3. $\boxed{27}$

4. (a) $\boxed{8099}$ (b) $\boxed{25}$ (c) $\boxed{1596}$

5. Let m and n our two counting numbers. We want their squares (m^2 and n^2) to differ (subtraction) by 15, or $m^2 - n^2 = 15$. We can factor the first part as $(m-n)(m+n) = 15$. Since m and n are counting numbers, $m - n$ and $m + n$ are counting numbers. So we need the whole factors of 15: 1 and 15, 3 and 5.

   First consider for $m - n = 1$ and $m + n = 15$. From our sum and difference lecture, we know $m = 8$ and $n = 7$.

   Now consider for $m - n = 3$ and $m + n = 5$. Then $m = 4$ and $n = 1$.

   Since we are squaring the numbers, it doesn't matter if m and n are positive or negative, so all of our answers are $\boxed{(\pm 4, \pm 1), (\pm 8, \pm 7)}$

6. First we manipulate $\frac{1}{m} + \frac{1}{n} = \frac{1}{5}$ so it's easier to work with.

$$\begin{aligned}
\frac{1}{m} + \frac{1}{n} &= \frac{1}{5} \\
\frac{n+m}{nm} &= \frac{1}{5} \text{ (adding fractions)} \\
5n + 5m &= nm \text{ (cross multiplying)} \\
5n + 5m - nm &= 0 \text{ (subtract both sides by nm)} \\
5n - nm + 5m - 25 &= -25 \text{ (subtract 25 from both sides)} \\
n(5-m) - 5(5-m) &= -25 \text{ (factor by grouping)} \\
(n-5)(5-m) &= -25 \text{ (factor by grouping)} \\
(n-5)(m-5) &= 25 \text{ (multiply -1 to both sides)}
\end{aligned}$$

   This last equation is possible if $n - 5 = 5$ and $m - 5 = 5$, if $n - 5 = 1$ and $m - 5 = 25$, or if $n - 5 = 25$ and $m - 5 = 1$. So the possible values for m are 10, 30, and 6. So the sum of possible values for m is $\boxed{46}$

7. In order for a perfect square to have a multiple of 11, it must have a multiple of 11^2. So we will solve the equation below for n:

$$99 < (11n)^2 < 1000$$

I chose these end numbers because we want 3 digit numbers. First square root all sides of the inequality: $9.9 < 11n < 31.6$. Now divide all sides by 11: $.9 < n < 2.9$. How many whole number n's are between these two values? 2, for when $n = 1, 2$. $\boxed{2}$ The two three-digit perfect squares that are multiples of 11 are 121 and 484.

8. How many numbers less than 100 are multiples of 2? $\lfloor \frac{99}{2} \rfloor = 49$ (the brackets mean to round down)

   How many numbers less than 100 are multiples of 3? $\lfloor \frac{99}{3} \rfloor = 33$

   How many numbers less than 100 are multiples of both 2 and 3? $\lfloor \frac{99}{6} \rfloor = 16$

   Recall from the Venn Diagram lecture: there are $49 + 33 - 16 = \boxed{66}$

9. $LCM(6, 12, 15) = 60$. So find how many multiples of 60 are less than 1000: $\lfloor \frac{1000}{60} \rfloor = \boxed{16}$

10. Problem was ungraded. $\boxed{49{,}500{,}000}$