



Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 17.2 - Units Digits

Name:

Date:

1. $\boxed{0}$

2. $\boxed{3}$

3. First, we will find the units digit of each subtraction:

$$(972 - 268) \times (973 - 267) \times (974 - 266) + (968 - 272) \times (967 - 273) \times (966 - 274) - (999 - 222)^3 \\ = (4) \times (6) \times (8) + (6) \times (4) \times (2) - (7)^3$$

Now we find the units digit of each multiplication:

$$= 2 + 8 - 3$$

Finally, we compute the units digit is $\boxed{7}$

4. $\boxed{0}$

5. First, make sure you read the question right: *What is the last two digits of the product of all ODD numbers less than 40?* Many of you put 00 as the answer, meaning you read the question as all the numbers less than 40.

Now we must consider all these odd numbers. One number we like to deal with is 5, because the last two digits of 5's never change. Notice $5^1 = 5, 5^2 = 25, 5^3 = 125, 5^4 = 625$. The last two digits are always 25. Since we are multiplying 5, 15, 25, and 35 in this product, we will have plenty of 5's to make our pattern - in fact, we will have 5 5's. Also notice that no matter what odd number you multiply to a 5-ending number, the units digit is still 5. For instance, $17 \times 5 = 85$. Now let's look at what happens when you multiply an odd number by 125: $43 \times 125 = 5375$, or $125 \times 27 = 3375$. The last two digits are always 75. So we can deduce that the last two digits will be $\boxed{75}$. (the answer key was wrong when the homework as graded. If you would like a point back, please see the TA).

6. $\boxed{7}$

7. For those who don't know, factorials, or the ! after numbers, mean to multiply all the numbers below. So $4! = 4 \times 3 \times 2 \times 1$, $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, and so on. So this expression could be very beastly, but lets look at the first few terms to see if we can get a hint how to solve the problem.

$$1! = 1$$

$$2! = 2 \times 1 = 2$$

$$3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Finally, we've reached a term with a 0 in the units place. Each factorial greater than 5! will also have a 0 in the units place, because each multiplication will have a $5 \times 2 = 10$ in it. So all we need to do to find the units digit of the sum is look at the first 4 terms, because the rest will have units digit 0. So $1 + 2 + 6 + 4 = 13$, so the units digit is $\boxed{3}$



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8.
9.
10.