Let's recall a few definitions Dr. Li gave in class. Let's say set $A = \{1, 2, 3, 4, 5\}$ and set $B = \{2, 4, 6, 8, 10\}$. Just think of sets as buckets of numbers.

When we say A *union* B, or $A \cup B$, this means the set formed by combining A and B. So $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$. Notice that 2 and 4 are in both sets, but we don't write them twice. We only write each number that appears in either set once. In a Venn Diagram, $A \cup B$ means all the elements present in the whole diagram.

When we say A *intersect* B or $A \cap B$, this means the set of elements in common with A and B. So $A \cap B = \{2, 4\}$. These are the two elements that both A and B have in common. In a Venn Diagram, $A \cap B$ means the intersection part.

When we say $|A|$, this means size of A, or how many elements are in the set. Here $|A| = 5$. Also, $|B| = 5$. $|A \cup B| = 8$, and $|A \cap B| = 2$.

Now we can write two important formulas:

$|A \cup B| = |A| + |B| - |A \cap B|$ (in other words, the number of all the things in a Venn Diagram total is the number of things in A plus the number of things in B minus the number of things in the middle intersection of the Venn diagram.)

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$.

1. $\boxed{17}$

2. $\boxed{50}$

3. $\boxed{34}$

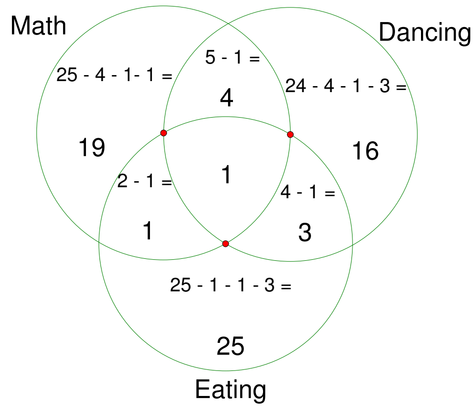
4. $\boxed{7}$

5. Problem omitted.

6. Method 1: slow way. We need to work backwards to solve this problem. We will start by placing the 1 student who joined all three clubs in the center of our Venn diagram.

   Now we will consider the double-club joiners. Notice the language of the problem: *5 students joined both the math and dancing clubs.* It does not say they ONLY joined math and dancing clubs. So this statement means that 5 students are in both math and dancing, but could possibly be in eating club as well. So we subtract the 1 student from the center of the Venn diagram from each of the given numbers, and place them in the appropriate places on our Venn Diagram. So this gives us exactly 4 students are in only M and D, 1 is only in M and E, and 3 are only in D and E.

   Finally, we move to the one-club numbers. We are given that there are 25 students in the math club, 24 students in the dancing club, and 30 students in the eating club. Again, these are not exclusive, so we have to subtract to find out how many are only in math club, only in dancing, and only in eating.



Now we add all the numbers in the Venn Diagram to get how many students joined clubs: $19 + 16 + 25 + 4 + 1 + 3 + 1 = \boxed{69}$

Method 2: inclusion-exclusion principle.

Using the formulas stated at the top of the page, we will write the information given in the problem mathematically. Let M be the number of people in math club, D be the number of people in dance club, and E be the number of people in eating club.

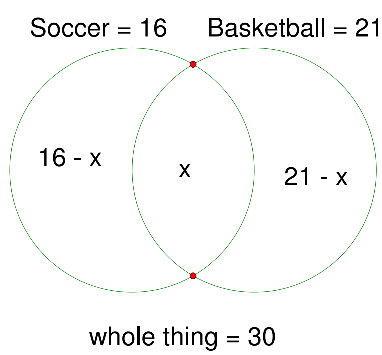
$|M| = 25, |D| = 24, |E| = 30.$ $|M \cap D| = 5, |D \cap E| = 4, |M \cap E| = 2.$ $|M \cap D \cap E| = 1.$

Now we'll use the formula for three sets on the top of the page:

$|M \cup D \cup E| = |M| + |D| + |E| - |M \cap D| - |M \cap E| - |D \cap E| + |M \cap D \cap E|$

$= 25 + 24 + 30 - 5 - 4 - 2 + 1 = \boxed{69}$

7. Method 1: slow way. Let's draw a Venn Diagram, labeling everything correctly. Careful: do NOT put 16 in the soccer part and 21 in the basketball part. We don't know what goes in those parts yet. We put 16 and 21 on the outside of the circles. Label the intersection x and write $16 - x$ and $21 - x$ in their respective places.



Now we can solve for x. We know the whole thing has to add up to 30, so we write the equation $(16 - x) + x + (21 - x) = 30$. Now simplify and solve: $16 + 21 - x + x - x = 30 \Rightarrow 37 - x = 30 \Rightarrow 37 - 30 = x = 7$. So $\boxed{7}$ play both sports. Now we can find how many only play soccer: $16 - x = 16 - 7 = \boxed{9}$. How many play only basketball: $21 - x = 21 - 7 = \boxed{14}$.

Method 2: inclusion-exclusion principle.

We are given $|S \cup B| = 30, |S| = 16, and|B| = 21$. Using the equation from the top of the page, we know $|S \cup B| = |S| + |B| - |S \cap B|$. Now we will solve for $|S \cap B|$, or the number of students in both sports: $30 = 16 + 21 - |S \cap B| \Rightarrow |S \cap B| = 37 - 30 = \boxed{7}$. To see how many only play soccer, subtract $|S| - |S \cap B| = 16 - 7 = \boxed{9}$, and to find how many play only basketball, subtract $|B| - |S \cap B| = 21 - 7 = \boxed{14}$.

8. Let A be the set of numbers from 1 to 1000 that are divisible by 2. Let B be the set of numbers that are divisible by 7. Then $A \cap B$ is the set of numbers that are divisible by BOTH 2 and 7, or are divisible by 14. Now let's find the sizes of each set.

   $|A| = \left\lfloor \frac{1000}{2} \right\rfloor = 500$ (the brackets mean "round down whatever is inside here", so $\lfloor 4.7 \rfloor = 4$).

   $|B| = \left\lfloor \frac{1000}{7} \right\rfloor = 142$

   $|A \cap B| = \left\lfloor \frac{1000}{14} \right\rfloor = 71$

   The formula above gives us $|A \cup B| = |A| + |B| - |A \cap B|$. So we find $|A \cup B = 500 + 142 - 71 = \boxed{571}$

9. Let A be the set of three digit numbers that are divisible by 2. Let B be the set of three digit numbers that are divisible by 3. Let C be the set of three digit numbers that are divisible by 3. Then $A \cap B$ is the set of numbers that are divisible by BOTH 2 and 3, or are divisible by 6. Also, $B \cap C$ is the set of numbers that are divisible by BOTH 3 and 5, or are divisible by 15. And $A \cap C$ is the set of numbers that are divisible by BOTH 2 and 5, or are divisible by 10. Finally, $A \cap B \cap C$ is the set of numbers that are divisible by 2, 3, AND 5, or are divisible by 30. Now let's find the sizes of each set.

   $|A| = \lfloor \frac{999}{2} \rfloor - \lfloor \frac{99}{2} \rfloor = 499 - 49 = 450$

   (I have to do the subtraction because we only want 3 digit numbers, so I have to subtract out the two digit numbers that are divisible by 2.)

   $|B| = \lfloor \frac{999}{3} \rfloor - \lfloor \frac{99}{3} \rfloor = 333 - 33 = 300$

   $|C| = \lfloor \frac{999}{5} \rfloor - \lfloor \frac{99}{5} \rfloor = 199 - 19 = 180$

   $|A \cap B| = \lfloor \frac{999}{6} \rfloor - \lfloor \frac{99}{6} \rfloor = 166 - 16 = 150$

   $|B \cap C| = \lfloor \frac{999}{15} \rfloor - \lfloor \frac{99}{15} \rfloor = 66 - 6 = 60$

   $|A \cap C| = \lfloor \frac{999}{10} \rfloor - \lfloor \frac{99}{10} \rfloor = 99 - 9 = 90$

   $|A \cap B \cap C| = \lfloor \frac{999}{30} \rfloor - \lfloor \frac{99}{30} \rfloor = 33 - 3 = 30$

   Well, that was annoying. Now let's use the formula from the top of the page to get the answer:

   $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

   $= 450 + 300 + 180 - 150 - 60 - 90 + 30 = \boxed{660}$

10. $\boxed{8}$