



Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 11.1 - Algebra Equations

Name:

Date:

1. $l = 6 + 2w$

2. Let's make some algebraic sentences based off of the information in the problem. Let d = number of dimes, n = number of nickels, and q = number of quarters.

The number of dimes is twice the number of nickels. $d = 2n$

The number of quarters is 6 more than the number of nickels. $q = 6 + n$

Matthew has 46 coins amounting to \$6.50. $.25q + .10d + .05n = 6.50$

Now we solve. Replace the d in the third equation with $2n$ (since the first equation tells us they are equal). Replace q in the third equation with $6 + n$ (since the second equation tells us they are equal). This method is called substitution. So now we have $.25(6 + n) + .10(2n) + .05n = 6.50$. Now simplify: $.25 \times 6 + .25n + .20n + .05n = 6.50 \Rightarrow 1.50 + .25n + .20n + .05n = 6.50$. Now we can add all the terms with n in them: $1.50 + .50n = 6.50$. Subtract 1.50 from both sides of the equation: $.50n = 5.00$. Now divide both sides by .50: $n = 10$.

So we know he has $n = 10$ nickels. Now we can use our previous equations to find out d and q . Since $d = 2n$, then $d = 2(10) = 20$, and since $q = 6 + n$, then $q = 6 + 10 = 16$.

$$n = 10, d = 20, q = 16$$

3. Use the strategy from above. Let J equal the money of Jae, C be the money of Connor, and M be the money of Mitchell. Set up equations: $J = 2C$, $C = 3M$, $J + C + M = 357$. Look at $J = 2C$ and $C = 3M$. Replace the C in the first equation with $3M$: $J = 2(3M) = 6M$. Now replace all the letters in the last equation: $J + C + M = (6M) + (3M) + M = 357$. Now simplify: $10M = 357$. Divide both sides by 10: $M = 35.7$. So Mitchell has \$35.70. Now we can find J and C : $J = 6M = 6(35.7) = 214.20$, $C = 3M = 3(35.7) = 107.10$.

$$M = \$35.70, C = \$107.10, J = \$214.20$$

4. $154, 77$

5. $l = 30, w = 10$

6. $l = w - 6$. $p = 2w + 2l = 60$. Use substitution: $2w + 2(w - 6) = 60 \Rightarrow 2w + 2w - 12 = 4w - 12 = 60$. Add 12 to both sides: $4w = 72$. Divide both sides by 4: $w = 18$. Now find l : $l = (18) - 6 = 12$. $18, 12$

7. $S = \$7,000, F = \$22,000$

8. $f = 10 + s$. If they traveled 272 miles away from each other in 8 hours, the average speed was $272 \div 8 = 34$ naut. mph. The combined speed of the two boats is $f + s$, so we can write the equation $f + s = (10 + s) + s = 10 + 2s = 34$. Subtract 10 from both sides: $2s = 24$. Divide both sides by 2: $s = 12$. So the slower boat goes 12 nautical mph, and the faster boat goes $10 + 12 = 22$ nautical mph. So how far did each go in 8 hours? Multiply: slow boat went $12 \text{ mph} \times 8 \text{ hours} = 96$ nautical miles, fast boat went $22 \text{ mph} \times 8 \text{ hours} = 176$ nautical miles. (check: they add up to 272? $176 + 96 = 272$.) $\text{fast: } 176, \text{ slow: } 96$

9. The perimeter of the small square (with side s) is $4s$. The perimeter of the larger square (with side l) is $4l$. We know that the side length of the larger square is twice the smaller,



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so $l = 2s$. So we find the perimeter of the larger square is $4l = 4(2s) = 8s$. So the sum of the perimeters is $4s + 8s = 12s = 648$. Divide both sides by 12: $s = 54$. The length of the smaller square is 54, the length of the larger square is $2 \times 54 = 108$. $\boxed{54, 108}$

10. Set up equations where $w = \$$ of wife, $s = \$$ of son, and $d = \$$ of daughter. $w + s + d = 75000$, $s = 3d$, $w = s + d = (3d) + d = 4d$. Use substitution: $w + s + d = 4d + 3d + d = 8d = 75000$. So when we divide both sides by 8, we get that $d = \$9375$. Then $s = 3d = 3(9375) = 28125$, and $w = 4d = 4(9375) = 37,500$. $\boxed{d = \$9,375, s = \$28,125, w = \$37,500}$