



Math Olympiad and Problem Solving Programs  
E210 - Introductory Math Competitions  
Problem Set 10.2 - Contest Number Theory

Name:

Date:

- Let's look at a few examples of the sums of two primes.  $7 + 13 = 20$ , and  $11 + 19 = 30$ . The sums aren't prime, because the two primes are  $odd + odd = even$ . So in order to have a prime sum, the sum MUST be odd. To have an odd sum, we must do  $even + odd = odd$ . Therefore, there has to be an even prime added to an odd prime. The only even prime is 2, so 2 must be one of the primes.  C
- Look at each choice:  
 $12^2 + 33^2 = 2^4 \cdot 3^2 + 3^2 \cdot 11^2 = 3^2(2^4 + 11^2)$ . So this number can be expressed as a product, so it cannot be prime.  
 $14^2 + 18^2 = even + even = even$ . Not prime.  
 $15^2 + 49^2 = odd + odd = even$ . Not prime.  
 $13^2 + 38^2 = odd + even = odd$ .  $13^2 + 38^2 = 13^2 + 2^2 \cdot 19^2$ , can't be factored.  $13^2 + 38^2 = 1613$ , test the primes less than 40, which are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37. None work, so 1613 is prime.  B
- If the perimeter is 36, then  $l + w = 18$ . So our two side lengths must add up to 18. Consider the combinations of prime number that add up to 18:  $5 + 13$ ,  $7 + 11$ . The largest area is the shape closest to a square, so  $7 \times 11 = 77$ .  77
- 43
- 270
- Prime factorize:  $391 = 17 \times 23$ , and  $713 = 23 \times 31$ . So their greatest common factor is  23.
- 31
- The list of all 2-digit multiples of 7: 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98. You can average them, or you can just choose the middle number of the list. Either way, you will get  56.
- 122
- The greatest common factor of positive integers  $m$  and  $n$  is 6. The least common multiple of  $m$  and  $n$  is 126. What is the least possible value of  $m + n$ ? (MATHCOUNTS)  60