



Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 9.1 - Counting Divisors

Name:

Date:

1. $\boxed{6}$
2. $\boxed{16}$
3. $\boxed{20}$
4. $\boxed{12}$
5. $(1 + 72) + (2 + 36) + (3 + 24) + (4 + 18) + (6 + 12) + (8 + 9) = \boxed{195}$
6. $\boxed{18}$
7. Since there are 11 factors, the exponents of the prime factorization of n can be written as m^{10} for some m that we don't know. So if we square n then we will get $(m^{10})^2 = m^{20}$. So there are $20 + 1 = \boxed{21}$ factors of n^2 .
8. This is a trick question! $n = 2^2 \cdot 3^3 \cdot 4^3 \cdot 6^2$ is not prime! So first prime factorize: $n = 2^2 \cdot 3^3 \cdot 4^3 \cdot 6^2 = 2^2 \cdot 3^3 \cdot 2^6 \cdot 2^2 \cdot 3^2 = 2^{10} \cdot 3^5$ So there are $(10 + 1)(5 + 1) = 11 \times 6 = \boxed{66}$ factors.
9. An even factor MUST have a factor of 2 in it. So move around the factors to make sure each number has a 2: $n = 2^2 \cdot 3^1 \cdot 7^2 = 2(2 \cdot 3 \cdot 7^2)$. Now count how many factors of the number in the parenthesis: $(1 + 1)(1 + 1)(2 + 1) = 2 \cdot 2 \cdot 3 = \boxed{12}$
10. Let's find all the divisors of 150: 1, 150, 2, 75, 3, 50, 5, 30, 6, 25, 10, 15. Now multiply them together: $(1 \cdot 150)(2 \cdot 75)(3 \cdot 50)(5 \cdot 30)(6 \cdot 25)(10 \cdot 15) = \boxed{150^6}$