



Math Olympiad and Problem Solving Programs
E210 - Introductory Math Competitions
Problem Set 8.2 - AMC 8 Counting Numbers

Name:

Date:

1. A
2. How many numbers between 1 and 46 are divisible by 3? $46 \div 3 = 15$ (we ignore the remainders for this problem). How many numbers are divisible by 5? $46 \div 5 = 9$. So there are 15 numbers divisible by 3 and 9 divisible by 5. But we've overcounted: we counted the numbers 15, 30, ... twice because they are both divisible by 3 and 5. So find how many numbers are divisible by 3×5 : $46 \div 15 = 3$. So we overcounted three times. So the number we are looking for is $15 + 9 - 3 = 21$. B
3. A
4. C
5. A
6. First count how many numbers have one 0 in the units place. Then there are 9 possibilities for the first digit and 9 possibilities for the tens digit. So there are $9 \times 9 = 81$ possible numbers with one zero in the units place. Now consider the case where the 0 is in the tens place. Then there are 9 possibilities for the number in the hundreds place and 9 possibilities for the number in the units place. So there are $9 \times 9 = 81$ possibilities ways to write a three digit number with one 0 in the tens place. So there are $81 + 81 = 162$ numbers of this form. D
7. If a number has 15, 20, and 25 as factors, then it must have $LCM(15, 20, 25)$ as a factor. We calculate the LCM to be 300. So we count how many multiples of 300 are between 1000 and 2000: 1200, 1500, 1800. There are 3 numbers. C
8. C
9. E
10. Let's count how many numbers up to 999 are divisible by 13: $999 \div 13 = 76$ (we ignore remainders on this method of counting). Now let's see how many of those multiples of 13 are two-digit numbers: $99 \div 13 = 7$. So there are 76 multiples of 13 up to 999, and 7 multiples of 13 up to 99, so we subtract $76 - 7 = 69$. C