Problem Set 8.1 - Perfect Squares

Name: Date:

1. (a) $\boxed{5625}$ (b) $\boxed{6400}$ (c) $\boxed{9025}$ (d) $\boxed{4225}$
2. (a) $\boxed{56}$ (b) $\boxed{31}$ (c) $\boxed{64}$ (d) $\boxed{72}$
3. (a) 2118 $\boxed{\text{not square}}$ (b) 729 $\boxed{\text{square}}$ (c) 3362 $\boxed{\text{not square}}$ (d) 1691 $\boxed{\text{not square}}$
4. $\boxed{4}$
5. A number that has exactly three factors has to be a square number of a prime number. For instance, $3^2 = 9$ has three factors, 1, 3, and 9. So to find the least two-digit square of a prime number, we will find the smallest square number that is the square of a prime. The first two-digit square number is 16, but 16 has 5 factors (1, 2, 4, 8, 16). The next largest two-digit square number is 25, which only has three factors: 1, 5, and 25. $\boxed{25}$
6. $\boxed{144}$
7. This problem again asks for three factors, and from problem 5, we determined that numbers with three prime factors are square numbers of primes. So we need to find how many square numbers of primes are less than 1000. Calculate $\sqrt{1000} = 31.622$. So we want all the prime numbers less than or equal to 31. These are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31. There are $\boxed{11}$ numbers.
8. The sum of two perfect squares CAN be another perfect square. This is called the Pythagorean Theorem, which gives a relationship for side lengths of a triangle (look it up on Wikipedia for more information). One example of this principle is $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. There are many more examples besides this.
9. $\boxed{1764}$
10. We want the smallest sibling perfect square that is also a perfect square, so we will start with the smallest possible square numbers. The smallest square numbers are 1, 4, and 9. Is 14 a perfect square? No. 19? No. 49? Yes. $\boxed{49}$