



Math Olympiad and Problem Solving Programs  
E210 - Introductory Math Competitions  
Problem Set 2.1 - Divisibility

Name:

Date:

1. Only (d) is divisible by 12.  d
2. In order for a number to be divisible by 4, its last two numbers must be divisible by 4. Think of all the two-digit numbers that you can make out of 1, 2, 3, and 4: 12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43. Of these numbers only 12, 24, and 32 are divisible by four. So we know the last two digits of our four digit number can be 12, 24, or 32. Now let's consider the first two digits of these numbers. For 12, the first two digits can be 43 or 34, so we can make 3412 and 4312. For 24, the first two digits can be 13 or 31, so we can make 1324 or 3124. For 32, the first two digits can be 14 or 41, so we can make 1432 and 4132. So we have made  6 four-digit numbers out of 1, 2, 3, and 4 that are divisible by 4.
3.  no
4. You got this problem wrong if you tried to prove by example. You can't prove something by example, because you only prove it's true for one particular case, instead of for all cases. So to prove for all examples, we first prime factorize 30:  $2 \times 3 \times 5$ . Now consider any five consecutive numbers. They could be 1, 2, 3, 4, 5, or they could be 72, 73, 74, 75, 76, and so on. We notice that every other number is even, so every two numbers has a factor of 2 in it. We also notice that every three numbers is a multiple of 3, so every three numbers has a factor of 3 in it. Also, we know that every five numbers is a multiple of 5, so every five numbers has a factor of 5 in it. So if we take any five numbers, we know that there must be an even number, there must be a multiple of three, and there must be a multiple of five. So their product is divisible by 30.
5. Let's see if there is a pattern of numbers that end with 2 and are divisible by 8. By guess-and-check, we find that the first 3 numbers that are divisible by 8 and end in 2 are 32, 72, and 112. We notice a pattern here: each number is +40 the number before it. Now we find the last number in this pattern. Consider 1000. We know 8 divides evenly into 1000. So take 8 away from 1000:  $1000 - 8 = 992$ . 992 ends with 2 and is divisible by 8, so it is the last number in our pattern. So we have 32, 72, 112, ..., 992. Now we must figure out how many numbers are in this sequence. First subtract the first and the last  $992 - 32 = 960$ , and divide by 40,  $960 \div 40 = 24$ . So we know we have added 40 24 times. Thus there are  $24 + 1 =$   25 numbers less than 1000 that end in 2 and are divisible by 8.
6.  21
7. The number  $54A$  is divisible by 6. A number that is divisible by 6 is divisible by 3 and by 2. So  $A$  has to be even so that the number is divisible by 2, and  $5 + 4 + A$  has to be divisible by 3. Since  $5 + 4$  is divisible by 3, we are just left with  $A$  having to be divisible by 3. So the only even numbers divisible by 3 are  0,6.
8.  no
9.  no
10. First, prime factorize 210:  $2 \times 3 \times 5 \times 7$ . We need a factor of 7 in our  $n!$ , and the first number that has a factor of 7 in it is 7. So the least possible value for  $n$  is  7.