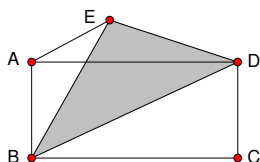


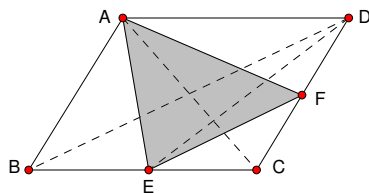
1. In the rectangle we have  $AB = 5, BC = AD = 8, \angle BAD = 90^\circ$ . This gives us  $[ABD] = \frac{1}{2} \cdot 5 \cdot 8 = 20$ . Since  $\triangle ABD$  and  $\triangle BDE$  share a common base,  $\overline{BD}$ , and altitude,  $[ABD] = [BDE] = \boxed{20}$ .



2. We can draw the diagonals to help us see the areas of the particular triangles as proportions of the parallelogram.

Since  $[ABCD] = 48$ , we know that  $[ABC] = [ACD] = [ABD] = [BCD] = 24$ . Now since  $E$  is the midpoint of  $\overline{BC}$ , we know that  $[ABE] = \frac{1}{2}[ABC] = 12$ . Also since  $F$  is the midpoint of  $\overline{CD}$ , we know that  $[ADF] = \frac{1}{2}[ACD] = 12$ .

Finding  $[CEF]$  is a little trickier, and for this we can draw  $\overline{DE}$ .  $[CDE] = \frac{1}{2}[BCD] = 12$  and  $[CEF] = \frac{1}{2}[CDE] = 6$ .

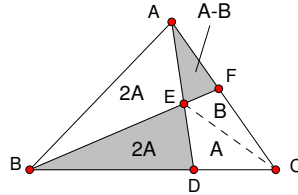


This gives us  $[AEF] = [ABCD] - [ABE] - [ADF] - [CEF] = 48 - 12 - 12 - 6 = \boxed{18}$ .

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3. We first draw  $\overline{CE}$ . Then, let  $[CDE] = A$ , and use proportions of triangle areas to get  $[BDE] = [ABE] = 2A$ . Let  $[CEF] = B$ . Then  $[AEF] = A - B$ .



Now notice that  $[ABD] = \frac{2}{3}[ABC] = \frac{10}{3} = 4A$ . This gives us  $A = \frac{5}{6}$ .

Lastly we use  $\overline{BF}$  as our common base to get the following proportions:

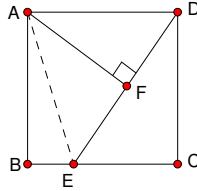
$$\begin{aligned} \frac{[ABE]}{[AEF]} &= \frac{[BCE]}{[CEF]} \\ \frac{2A}{A - B} &= \frac{3A}{B} \\ \frac{\frac{5}{3}}{\frac{5}{6} - B} &= \frac{\frac{5}{2}}{B} \\ \frac{5B}{3} &= \frac{25}{12} - \frac{5B}{2} \\ 20B &= 25 - 30B \\ 50B &= 25 \\ B &= \frac{1}{2} \end{aligned}$$

Now the shaded area consists of  $2A + A - B = 3A - B = 3\frac{5}{6} - \frac{1}{2} = \boxed{2}$ .

4. The side of square  $ABCD$  is 4. The length of  $DE$  is 5 and  $CE$  is 3. Find the length of  $AF$ .

$$\boxed{\frac{16}{5} = 3.2}$$

First we draw  $\overline{AE}$ . Now all of the triangles in the square are right triangles so we can use Pythagorean Theorem to find the missing lengths.



We know  $AB = 4$  and  $BE = 1$  since  $CE = 3$ . Pythagorean Theorem gives us the following:

$$(AE)^2 = 4^2 + 1^2$$

$$(AE)^2 = 16 + 1$$

$$(AE)^2 = 17$$

$$AE = \sqrt{17}$$

Now since  $AE = \sqrt{17}$ ,  $AD = 4$ ,  $DE = 5$ , we can use Pythagorean Theorem to find  $AF$ :

$$\sqrt{(AE)^2 - (AF)^2} + \sqrt{(AD)^2 - (AF)^2} = EF + DF = DE$$

$$\sqrt{17 - (AF)^2} + \sqrt{16 - (AF)^2} = 5$$

$$17 - (AF)^2 + 2\sqrt{[17 - (AF)^2][16 - (AF)^2]} + 16 - (AF)^2 = 25$$

$$33 - 2(AF)^2 + 2\sqrt{272 - 33(AF)^2 + (AF)^4} = 25$$

$$2\sqrt{272 - 33(AF)^2 + (AF)^4} = 2(AF)^2 - 8$$

$$\sqrt{272 - 33(AF)^2 + (AF)^4} = (AF)^2 - 4$$

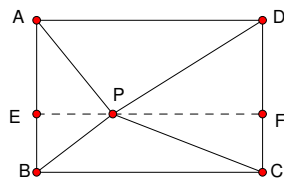
$$272 - 33(AF)^2 + (AF)^4 = (AF)^4 - 8(AF)^2 + 16$$

$$256 = 25(AF)^2$$

$$(AF)^2 = \frac{256}{25}$$

$$AF = \boxed{\frac{16}{5} = 3.2}$$

5. Let us first draw the altitudes of  $\triangle ABP$  and  $\triangle CDP$ , through  $P$ , perpendicular to  $\overline{AB}$  and  $\overline{CD}$ , intersecting at  $E, F$  respectively.



Now notice we get the following equations from just the areas of the triangles:

$$\begin{cases} \frac{1}{2}(AB)(PE) = 24 \\ \frac{1}{2}(CD)(PF) = 48 \end{cases}$$

But  $AB = CD$  so this really becomes:

$$\begin{cases} \frac{1}{2}(AB)(PE) = 24 \\ \frac{1}{2}(AB)(PF) = 48 \end{cases}$$

Now if we add these two equations together, we get the following:

$$\begin{aligned} \frac{1}{2}(AB)(PE) + \frac{1}{2}(AB)(PF) &= 24 + 48 \\ \frac{1}{2}(AB)(PE + PF) &= 72 \\ (AB)(PE + PF) &= 144 \end{aligned}$$

But  $PE + PF = BC$  so we really get  $(AB)(BC) = 144$ , which is the area of the rectangle  $ABCD$ .

This leaves the area of  $\triangle DAP$  to be  $[ABCD] - [ABP] - [BCP] - [CDP] = 144 - 24 - 20 - 48 = \boxed{52}$ .

6.  $\triangle ABF \sim \triangle CEF$ , giving us  $\frac{CE}{EF} = \frac{10}{BF}$ . But we know that  $BF + EF = 6$  so  $EF = 6 - BF$ . We now have the following proportion, simplified:  $(CE)(BF) = 60 - 10(BF)$ .

Now using  $\triangle BCF$ , which has an area of 6, we can come up with an equation using the area formula. With  $\overline{BF}$  as its base and  $\overline{CE}$  as its altitude,  $\frac{1}{2}(CE)(BF) = 6$  or  $(CE)(BF) = 12$ .

Now we can substitute  $(CE)(BF) = 12$  into the previous proportion to get  $12 = 60 - 10(BF)$ . Solving this gives us  $BF = 4.8$ . Then  $CE = 12 \div 4.8 = 2.5$ .

Finally we now know that  $DC = 10 + 2.5 = 12.5$  so the area of the trapezoid  $ABCD$  is  $\frac{10+12.5}{2} \cdot 6 = \boxed{67.5}$ .

