



Math Olympiad and Problem Solving Programs  
E130 - Honors Geometry Problem Solving  
Problem Set 24.2 - Similar Solids

Name:

Date:

**Notes (READ THIS FIRST):** If two measures of similar solids are in a ratio of  $a : b$ , their surface areas will be in a ratio of  $a^2 : b^2$  and their volumes will be in a ratio of  $a^3 : b^3$ . This will be applied to all of the following problems.

1. This might have been graded incorrectly. The ratio of the volumes of the entire cone to the part of the cone filled with water is  $8 : 1$ . This means that the measures of the cones will be in a ratio of  $2 : 1$ . The height of the cone is 4 cm so the height of the part filled with water must be  $\boxed{2 \text{ cm}}$ .

2.  $\boxed{10,000 \text{ cm}^3}$

3. The mass of an object is related to its volume so we will find the volume of the larger sphere. Since the measures of the spheres are in a ratio of  $4 : 5$ , their volumes must be in a ratio of  $64 : 125$ . This gives us the following proportion with their masses:

$$\frac{M}{120} = \frac{125}{64}$$
$$M = \boxed{234\frac{3}{8} = 234.375 \text{ kg}}$$

4. The bases are a part of the surface area so the ratio of the surface areas of the cones are  $9 : 16$ . This means that the ratio of their measures is  $3 : 4$  and the ratio of their volumes is  $27 : 64$ . This gives us the following proportion:

$$\frac{V}{448} = \frac{27}{64}$$
$$V = \boxed{189 \text{ cm}^3}$$

5. (a) The measures of the cylinders are in a ratio of  $5 : 4$  so their volumes are in a ratio of  $\boxed{125 : 64}$ .

(b) The measures of the cones are in a ratio of  $3 : 4$  their volumes are in a ratio of  $\boxed{27 : 64}$ .

(c) The measures of the spheres are in a ratio of  $2 : 3$  so their volumes are in a ratio of  $\boxed{8 : 27}$ .

6. The volumes of the two spheres are in a ratio of  $64 : 27$  so the ratio of their diameters, which are measures, is  $\boxed{4 : 3}$ .

7. The measures of the glasses have a ratio of  $2 : 3$  so their volumes have a ratio of  $8 : 27$ . This gives us the following proportion:

$$\frac{V}{54} = \frac{8}{27}$$
$$V = \boxed{16 \text{ cm}^3}$$

8. This problem may have been graded incorrectly. The volumes of the bottles are in a ratio of 28 : 75 so their measures are in a ratio of  $\sqrt[3]{28} : \sqrt[3]{75}$ . This gives us the following proportion:

$$\frac{h}{15} = \frac{\sqrt[3]{75}}{\sqrt[3]{28}}$$

$$h = \boxed{15 \sqrt[3]{\frac{75}{28}}}$$

9. The ratio of the volumes of the spheres is 4 : 1 so the ratio of their measures is  $\sqrt[3]{4} : 1$ . This gives us the following proportion:

$$\frac{r}{3} = \frac{\sqrt[3]{4}}{1}$$

$$r = \boxed{3 \sqrt[3]{4} \text{ cm}}$$

10. (a) The mass of an object is related to its volume so we will find the ratio of the volumes of the locomotive to its model. Since the ratio of their measures is 10 m : 40 cm = 1000 cm : 40 cm = 25 : 1, the ratio of their volumes is  $25^3 : 1^3 = 15625 : 1$ . This gives us the following proportion:

$$\frac{m}{72} = \frac{1}{15625}$$

$$m = \boxed{0.004608 \text{ tons} = 4.608 \text{ kg}}$$

- (b) Since the capacity of the tank is a measure of volume, we will continue to use our ratio 15625 : 1 for locomotive to model.

$$\frac{V}{0.85} = \frac{15625}{1}$$

$$V = \boxed{13281.25 \text{ L} = 13.28125 \text{ kL}}$$