Avid Academy for Gifted Youth

Math Olympiad and Problem Solving Programs

E130 - Honors Geometry Problem Solving

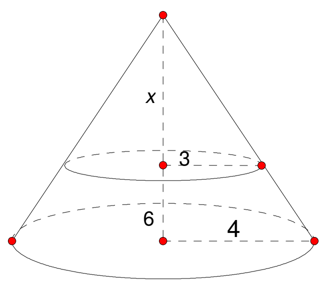
Problem Set 24.1 - Volumes and Surface Area

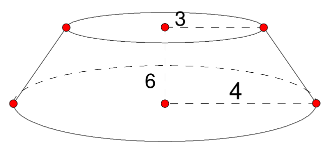
Name: Date:

1. $\boxed{\frac{250\pi}{3}}$

2. We have similar triangles in the cross section of the figure, giving the following proportion:

$$\begin{aligned} \frac{x}{3} &= \frac{x+6}{4} \\ 4x &= 3x + 18 \\ x &= 18 \end{aligned}$$





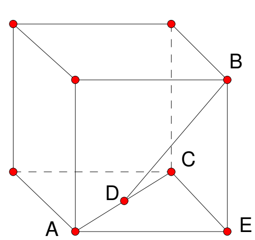
Now we can find the volume of the large cone and subtract from that the volume of the smaller cone to get the volume we are looking for:

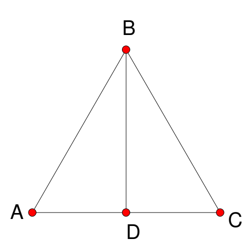
$$\begin{aligned} V &= \frac{1}{3} \cdot 4^2\pi \cdot 24 - \frac{1}{3} \cdot 3^2\pi \cdot 18 \\ &= \frac{\pi}{3}(24 \cdot 16 - 9 \cdot 18) \\ &= \frac{\pi}{3}(384 - 162) \\ &= \frac{\pi}{3}(222) \\ &= \boxed{74\pi} \end{aligned}$$

3. The tricky part of this problem is trying to picture the right angle produced by $\overline{BD} \perp \overline{AC}$. What we get from this is that $\angle BDC$ is a right angle.

$\triangle BAC$ is actually an equilateral triangle formed by diagonals of three sides of the square, and so $\overline{BD}$ is an altitude and median of $\triangle BAC$.

Since an edge of the cube is 8, a side of the equilateral triangle is $8\sqrt{2}$. We can easily deduce an altitude of $\triangle BAC$, which $\overline{BD}$ is, to be $\boxed{4\sqrt{6}}$.





4. $\boxed{A = 168, V = 112}$