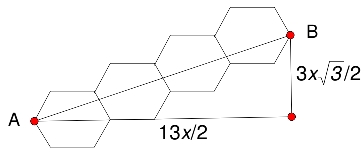
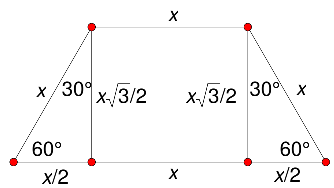
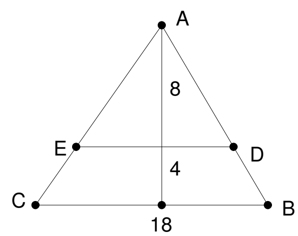
1. The simplest way to determine length, or distance, is to draw perpendiculars to form a right triangle. Now notice the legs of the triangle formed correspond to parts of the hexagon, which we must examine. I have drawn a diagram of the parts of the hexagon that are relevant, filling in all important information using 30-60-90 triangles.

   This gives us the legs of the right triangle to be $\frac{13x}{2}, \frac{3x\sqrt{3}}{2}$. Pythagorean Theorem gives us the hypotenuse, or the length of AB to be $\boxed{7x}$.





2. The diagram is shown below.



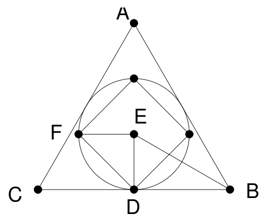
   Clearly we have two similar triangles: $\triangle ABC \sim \triangle ADE$. Using proportions, we can get the following:

$$\frac{DE}{18} = \frac{8}{12}$$
$$DE = 12$$

   Now the area of the trapezoid is equal to the area of $\triangle ABC$ minus the area of $\triangle ADE$.

$$\begin{aligned}[DECB] &= \frac{1}{2} \cdot 12 \cdot 18 - \frac{1}{2} \cdot 8 \cdot 12 \\ &= \frac{1}{2} \cdot 12(18 - 8) \\ &= 6 \cdot 10 = \boxed{60 \text{ cm}^2}\end{aligned}$$

3. A diagram of the described figure is shown below.



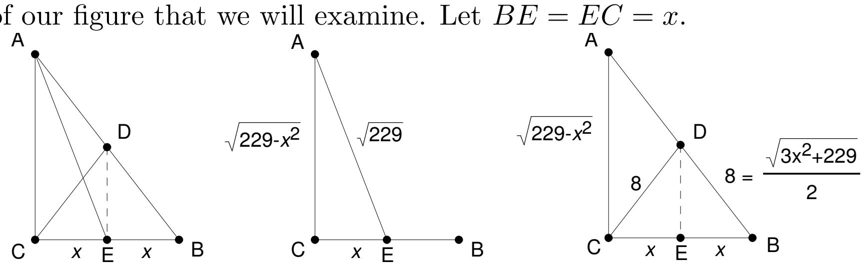
The center of the inscribed circle must be at the intersection of the altitudes/medians/angle bisectors of $\triangle ABC$. Notice this gives us $\triangle BDE$ to be 30-60-90. If we let $AB = AC = BC = x$ then $DB = \frac{x}{2}$ and then $ED = \frac{x\sqrt{3}}{6}$.

Notice now that $\overline{ED}$ is half of a diagonal of the square. We know that $\triangle DEF$ must be 45-45-90. This gives us one side of the square, $DF = \sqrt{2} \cdot \frac{x\sqrt{3}}{6} = \frac{x\sqrt{6}}{6}$. This gives us the area of the square to be $\left(\frac{x\sqrt{6}}{6}\right)^2 = \frac{x^2}{6}$.

The area of our triangle is much easier to find since its base is x and its height must be $\frac{x\sqrt{3}}{2}$. Thus the area is $\frac{1}{2} \cdot x \cdot \frac{x\sqrt{3}}{2} = \frac{x^2\sqrt{3}}{4}$.

Our final step is to find the ratio which is $\frac{x^2\sqrt{3}}{4} : \frac{x^2}{6} = \boxed{3\sqrt{3} : 2}$

4. The diagram is shown below. For the sake of ease of explanation, I also diagrammed two sections of our figure that we will examine. Let $BE = EC = x$.



First look at $\triangle ACE$. Using Pythagorean Theorem, we can get $AC = \sqrt{229 - x^2}$, as shown in the middle diagram.

Now if we look at the far right diagram, we can use Pythagorean Theorem again to get $AB = \sqrt{3x^2 + 229}$ and since CD is a median, $BD = \frac{\sqrt{3x^2+229}}{2}$.

Next draw $\overline{DE}$. Notice that it is a midline, a segment connecting two midpoints of a triangle. A property of this segment that we can use is that it must be parallel with the third side of the triangle, which is $\overline{AC}$. This means that $\overline{DE} \perp \overline{BC}$. Furthermore, since $\overline{DE}$ is now an altitude AND a median of $\triangle BCD$, $\triangle BCD$ must be isosceles. This means that $CD = BD = 8$.

Finally this gives us an equation we can use to solve for x:

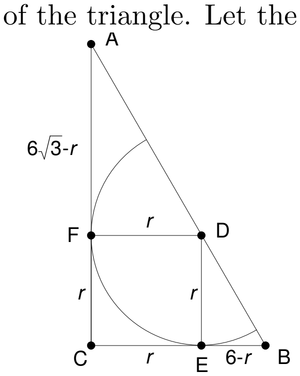
$$\begin{aligned}\frac{\sqrt{3x^2+229}}{2} &= 8\\ \sqrt{3x^2+229} &= 16\\ 3x^2+229 &= 256\\ 3x^2 &= 27\\ x^2 &= 9\\ x &= 3\end{aligned}$$

The length of the shorter leg, $BC = 2x = 2 \cdot 3 = \boxed{6}$.

5. $\boxed{15^\circ}$

6. This problem may have been graded incorrectly.

A diagram of the figure is shown below. Notice first that from the information that the hypotenuse is 12 and one leg is 6 that the triangle MUST be a 30-60-90 triangle. This gives the length of the longer leg to be $6\sqrt{3}$. The center of the circle being D, we can draw two radii from the center to the legs of the triangle. Let the radius be of length r.



Since $\overline{AC}$ and $\overline{BC}$ are tangent to the circle, the radii drawn must be perpendicular to their respective legs. Consequently, we form a square with side length r.

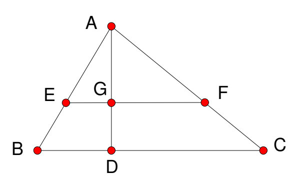
This now leaves $AF = 6\sqrt{3} - r$ and $BE = 6 - r$. Also we can show that $\triangle ADF \sim \triangle DBE$.

The similar triangles will give us a proportion we can use to solve for r:

$$\begin{aligned}\frac{AF}{FD} &= \frac{DE}{EB}\\ \frac{6\sqrt{3}-r}{r} &= \frac{r}{6-r}\\ 36\sqrt{3}-6r-6\sqrt{3}r+r^2 &= r^2\\ 36\sqrt{3}-(6+6\sqrt{3})r &= 0\\ 36\sqrt{3} &= (6+6\sqrt{3})r\\ r &= \frac{36\sqrt{3}}{6+6\sqrt{3}}\\ r &= \frac{6\sqrt{3}}{1+\sqrt{3}}\\ r &= \frac{6\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}\\ r &= 9-3\sqrt{3}\end{aligned}$$

Now the area of the semicircle is $(9-3\sqrt{3})^2\pi\cdot\frac{1}{2} = \boxed{(54-27\sqrt{3})\pi}$.

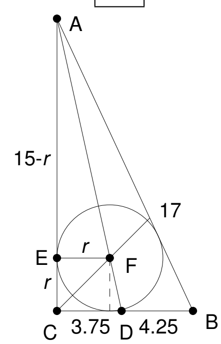
7. $\triangle AEF \sim \triangle ABC$. Since $[ABC]:[AEF] = 9:4$, we know that $AD:AG = BC:EF = 3:2$. This gives us $EF = 16$ and $AG = 4$. Now the area of $\triangle AEF$ is $\frac{1}{2}\cdot 4\cdot 16 = 32$. Using the ratio of the areas, the area of $\triangle ABC$ is 72. This gives us the area of the trapezoid to be $72-32 = \boxed{40}$.



8. The diagram is shown below. Using the Angle Bisector Theorem, we get $\frac{AC}{AB} = \frac{CD}{DB}$. This gives us $CD = 3.75, DB = 4.25$. Now we draw a radius from F to $\overline{AC}$, intersecting $\overline{AC}$ at E. This radius is perpendicular to $\overline{AC}$ and is thus parallel to $\overline{BC}$. $EC = r$ and $AE = 15 - r$. We also have similar triangles $\triangle AEF \sim \triangle ACD$. This gives us the following proportion:

$$\begin{aligned} \frac{AE}{EF} &= \frac{AC}{CD} \\ \frac{15-r}{r} &= \frac{15}{3.75} \\ 15r &= 56.25 - 3.75r \\ 18.75r &= 56.25 \\ r &= 3 \end{aligned}$$

Clearly now the area of the circle is $3^2\pi = \boxed{9\pi}$.



9. $\boxed{\frac{5\sqrt{3}}{4}}$

10. The information given is filled in the diagram below. I drew radius $\overline{MO}$ and radius $\overline{NQ}$ which both must be perpendicular to $\overline{MND}$. This gives us two similar triangles, $\triangle DMO \sim \triangle DNQ$. This gives us the following proportion:

$$\begin{aligned} \frac{BQ}{NQ} &= \frac{BO}{MO} \\ \frac{BD+1}{1} &= \frac{BD+1+1+3}{3} \\ BD+1 &= \frac{BD+5}{3} \\ 3BD+3 &= BD+5 \\ 2BD &= 2 \\ BD &= \boxed{1} \end{aligned}$$