Math Olympiad and Problem Solving Programs

E130 - Honors Geometry Problem Solving

Problem Set 19.1 - Circles

Name: Date:

1. $\boxed{\frac{4-\pi}{2} = 2 - \frac{\pi}{2}}$

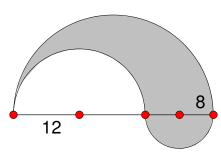
2. $\boxed{119}$

3. This is simply a matter of finding the area of the large semi-circle, subtracting from that the area of the medium semi-circle, and adding the area of the small semi-circle.

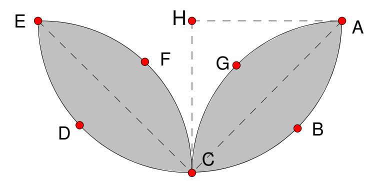
   Because the small and medium semi-circles have radii of 8 and 12, we can find the diameter of the large semi-circle to be 40, and its radius to be 20. Now we only need to find the areas:

$$\begin{aligned}\frac{20^2\pi}{2} - \frac{12^2\pi}{2} + \frac{8^2\pi}{2} &= \frac{400\pi}{2} - \frac{144\pi}{2} + \frac{64\pi}{2} \\ &= 200\pi - 72\pi + 32\pi \\ &= \boxed{160\pi}\end{aligned}$$

Problem 3



Problem 4



4. Because $ABCDE$ is the arc of a semi-circle, and AGC, CFE are arcs of two identical quadrants, we have a symmetry. If we draw segment $\overline{AC}$, the area formed by ABC is $\frac{1}{4}$ the total area we are looking for.

   Letting H be the center of the semi-circle, the area enclosed by $ABCH$ is $\frac{1}{4}$ of the circle so we know that $[ABCH] = \frac{1}{4}28^2\pi = 196\pi$.

   If we subtract off the area of $\triangle ACH$, we get the area of ABC. $[ABC] = 196\pi - \frac{1}{2}28^2 = 196\pi - 392$.

   Now we multiply by 4 to get the area of the entire shaded region: $4(196\pi - 392) = 784\pi - 1568 = 784\frac{22}{7} - 1568 = 2464 - 1568 = \boxed{896 \text{ cm}^2}$.

5. This problem may have been graded incorrectly.

   Since the radii are in a ratio of 2:1.5:0.5, and we are looking for the ratio of two areas, we can simply let the ratios of the circles be 2, 1.5, and 0.5.

   The area of the shaded region is the area of the smallest circle plus the area of the smallest semi-circle plus the area of the largest semi-circle minus the area of the medium semi-circle:

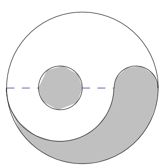
$$\begin{aligned} 0.5^2\pi + \frac{0.5^2\pi}{2} + \frac{2^2\pi}{2} - \frac{1.5^2\pi}{2} &= 0.25\pi + 0.125\pi + 2\pi - 1.125\pi \\ &= 1.25\pi \end{aligned}$$

   The area of the unshaded region is the area of the largest semi-circle minus the area of twice the smallest semi-circle plus the area of the medium semi-circle minus the area of the smallest semi-circle:

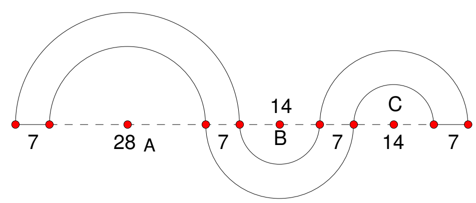
$$\begin{aligned} \frac{2^2\pi}{2} - 2\left(\frac{0.5^2\pi}{2}\right) + \frac{1.5^2\pi}{2} - \frac{0.5^2\pi}{2} &= 2\pi - 0.25\pi + 1.125\pi - 0.125\pi \\ &= 2.75\pi \end{aligned}$$

   The ratio of the shaded area to the unshaded area is therefore $1.25\pi : 2.75\pi = \boxed{5 : 11}$

Problem 5



Problem 6



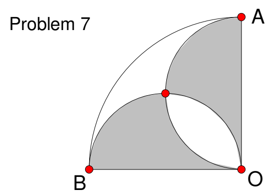
6. This problem may also have been graded incorrectly.

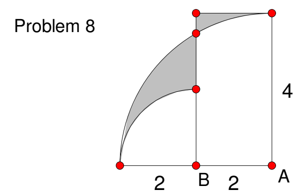
   This problem is simply a matter of subtracting the smaller semi-circles from the larger semi-circles.

$$\begin{aligned} \frac{21^2\pi}{2} - \frac{14^2\pi}{2} + \frac{14^2\pi}{2} - \frac{7^2\pi}{2} + \frac{14^2\pi}{2} - \frac{7^2\pi}{2} &= 220.5\pi - 98\pi + 98\pi - 24.5\pi + 98\pi - 24.5\pi \\ &= 296.5\pi \\ &= 296.5\left(\frac{22}{7}\right) \\ &= \boxed{847} \end{aligned}$$

7. The points mentioned in this problem were not labelled correctly in the picture.

   In the following figure, OA and OB are diameters of the two semi-circles. $OA = OB = 6$ and $\angle AOB = 90°$. Find the area of the shaded region. $\boxed{18}$

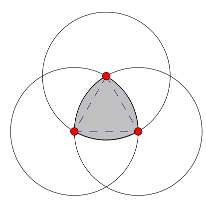




8. The points mentioned in this problem were not labelled correctly in the picture.

   A and B are the centers of two circles. Find the difference of the two shaded regions. $\boxed{3\pi - 8}$

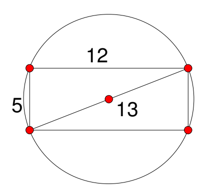
9. The picture looks like the following:



   Notice that if we connect the centers of the circles, we get an equilateral triangle with side length 12. This shows that in each of the circles, the sector formed by the arc is 60°. Now the shaded area is really the same as three of these 60° sectors minus twice the area of the equilateral triangle:

$$3\left(\frac{60}{360}\cdot 12^2\pi\right) - 2\left(\frac{1}{2}\cdot 12\cdot 6\sqrt{3}\right) = \frac{1}{2}\cdot 144\pi - 72\sqrt{3}$$
$$= \boxed{72\pi - 72\sqrt{3}}$$

10. The picture looks like the following:



   Each diagonal of the rectangle is actually a diameter of the circle. Using Pythagorean Theorem, we can find the diameter of the circle to be 13 and its radius to be $\frac{13}{2}$. Then the area of the region we're looking for is simple:

$$\left(\frac{13}{2}\right)^2\pi - 12(5) = \boxed{\frac{169\pi}{4} - 60}$$