1. (a) $\boxed{\frac{1}{8}}$

   (b)

$$\begin{aligned}\frac{(27^{\frac{1}{3}})^{\frac{5}{2}}}{27^{-\frac{1}{2}}} &= \frac{27^{\frac{5}{6}}}{27^{-\frac{1}{2}}}\\ &= 27^{\frac{5}{6}-(-\frac{1}{2})}\\ &= 27^{\frac{4}{3}}\\ &= (3^3)^{\frac{4}{3}}\\ &= 3^4 = \boxed{81}\end{aligned}$$

   (c)

$$\begin{aligned}\left(\frac{1}{2}\right)^5 \times 4^{1.5} + \left(\frac{8}{27}\right)^{-\frac{2}{3}} &= \frac{1^5}{2^5} \times (2^2)^{1.5} + \left(\frac{3^3}{2^3}\right)^{\frac{2}{3}}\\ &= \frac{2^3}{2^5} + \frac{3^2}{2^2}\\ &= \frac{1}{2^2} + \frac{9}{2^2}\\ &= \frac{10}{4} = \boxed{\frac{5}{2}}\end{aligned}$$

2. (a)

$$\begin{aligned}\left(a^{\frac{1}{2}}b^2\right)^{\frac{3}{4}} \times \left(ab^{-8}\right)^{\frac{1}{8}} &= \left(a^{\frac{3}{8}}b^{\frac{3}{2}}\right) \times \left(a^{\frac{1}{8}}b^{-1}\right)\\ &= a^{(\frac{3}{8}+\frac{1}{8})}b^{(\frac{3}{2}-1)}\\ &= a^{\frac{1}{2}}b^{\frac{1}{2}}\\ &= (ab)^{\frac{1}{2}}\\ &= \boxed{\sqrt{ab}}\end{aligned}$$

   (b)

$$\begin{aligned}\sqrt[3]{8p^6} \div \left(16p^{-2}\right)^{\frac{1}{2}} &= 2p^2 \div 4p^{-1}\\ &= 2p^2 \div \frac{4}{p}\\ &= 2p^2 \times \frac{p}{4}\\ &= \frac{2p^3}{4} = \boxed{\frac{p^3}{2}}\end{aligned}$$

3. Expand and simplify the following:

   (a)

$$\begin{aligned}(x+2y)(x-2y)^2 &= (x^2-4y^2)(x-2y)\\ &= \boxed{x^3-2x^2y-4xy^2+8y^3}\end{aligned}$$

   (b) $\boxed{12bc}$

4. (a) $\boxed{-2(2x+1)(x-7)}$

   (b)

$$\begin{aligned}4-4a^2+a^2b^2-b^2 &= 4(1-a^2)-b^2(-a^2+1)\\ &= (4-b^2)(1-a^2)\\ &= \boxed{(1+a)(1-a)(2+b)(2-b)}\end{aligned}$$

5. $\boxed{\frac{16}{63}}$

6. $\boxed{\frac{1}{x}}$

7. We have a lowest common denominator of $x(x-1)(x+2)$:

$$\begin{aligned}\frac{3}{x(x-1)}-\frac{x+3}{(x-1)(x+2)} &= \left(\frac{3}{x(x-1)}\right)\left(\frac{x+2}{x+2}\right)-\left(\frac{x+3}{(x-1)(x+2)}\right)\left(\frac{x}{x}\right)\\ &= \frac{3x+6-(x^2+3x)}{x(x-1)(x+2)}\\ &= \frac{3x+6-x^2-3x}{x(x-1)(x+2)}\\ &= \boxed{\frac{6-x^2}{x(x-1)(x+2)}}\end{aligned}$$

   You can simplify the denominator if you desire.

8. (a)

$$\frac{-\left(-\frac{9}{4}\right) \pm \sqrt{\left(-\frac{9}{4}\right)^2 - 4(1)\left(-\frac{2}{3}\right)}}{2(1)} = \frac{\frac{9}{4} \pm \sqrt{\frac{81}{16} + \frac{8}{3}}}{2}$$
$$= \frac{\frac{9}{4} \pm \sqrt{\frac{371}{48}}}{2}$$
$$= \frac{\frac{9}{4} \pm \frac{\sqrt{371}}{4\sqrt{3}}}{2}$$
$$= \frac{9}{8} \pm \frac{\sqrt{371}}{8\sqrt{3}}$$
$$= \frac{9\sqrt{3} \pm \sqrt{371}}{8\sqrt{3}}$$
$$= \boxed{\frac{27 \pm \sqrt{1113}}{24}}$$

(b) $\boxed{x = \frac{-1 \pm \sqrt{41}}{4}}$

9. Let t be the time in hours it took him to travel 30 km at x km/h. Then $xt = 30$ or $t = \frac{30}{x}$. Our second equation is given by the journey if he reduces his speed by 10 km/h: $(x-10)(t+\frac{2}{15}) = 30$. Notice that instead of 8 minutes we must convert to $\frac{2}{15}$ hours. We substitute $t = \frac{30}{x}$ into the second equation to get the following result:

$$(x-10)\left(\frac{30}{x} + \frac{2}{15}\right) = 30$$
$$30 - \frac{300}{x} + \frac{2x}{15} - \frac{4}{3} = 30$$
$$-\frac{300}{x} + \frac{2x}{15} - \frac{4}{3} = 0$$
$$-4500 + 2x^2 - 20x = 0$$
$$x^2 - 10x - 2250 = 0$$

Solve using the Quadratic Formula to get $\boxed{x = \frac{10+5\sqrt{10}}{2} \approx 12.91}$.

10. $(2x-1)$ is the height of our trapezoid with bases $(3x-8)$ and $(5x-6)$. Using the information that the area of our trapezoid is 31 units squared, we get the following equation:

$$\begin{aligned}\frac{(3x-8)+(5x-6)}{2}\times(2x-1)&=31\\ \frac{8x-14}{2}\times(2x-1)&=31\\ (4x-7)\times(2x-1)&=31\\ 8x^2-18x+7&=31\\ 8x^2-18x-24&=0\\ 4x^2-9x-12&=0\end{aligned}$$

When we solve this using the Quadratic Formula, we get $x=\frac{9+\sqrt{273}}{8}$. Note that the negative value is not possible. Plug this into $(5x-6)$ to get PS:

$$\begin{aligned}5\left(\frac{9+\sqrt{273}}{8}-6\right)&=\frac{45+5\sqrt{273}}{8}-\frac{48}{8}\\ &=\boxed{\frac{-3+\sqrt{273}}{8}\approx 1.69}\end{aligned}$$