



Name:

Date:

Instruction: You may use a calculator to solve these problems.

1. $\boxed{3\sqrt{2}, 2\sqrt{6}, 5, \sqrt{27}}$

2. $\boxed{14}$

3. Suppose our digits are A, B, C then our number can be represented as $N = 100A + 10B + C = 4ABC$. We need to use our deduction skills to examine this information to narrow down our possibilities and then do some guessing and checking.

(1) First of all, notice that this means that our number N has to be divisible by 4. In particular this helps us because this tells us $10B + C$ has to be divisible by 4.

Since our first bit of information tells us about the last two digits, let A depend on B, C by solving for A :

$$\begin{aligned}100A + 10B + C &= 4ABC \\10B + C &= 4ABC - 100A \\10B + C &= A(4BC - 100) \\ \frac{10B + C}{4BC - 100} &= A \\ \frac{10B + C}{4(BC - 25)} &= A\end{aligned}$$

(2) To further narrow down B, C , we can say that $BC - 25$ divides $10B + C$.

(3) The last bit of information that we can deduce is that since N is a three-digit number, $100 \leq 4ABC < 1000$ or $25 \leq BC < 250$ (since A is some positive integer).

Now let's begin looking at our number $10B + C$, which must be a multiple of 4. I will only go through the first few multiples of 4 and let you check the rest up to the answer.

04, 08, 12, ..., 44: $BC < 25$ and does not meet requirement 3

48: $BC - 25 = 32 - 25 = 7$ does not divide $10B + C = 48$ and does not meet requirement 2

52, 56, 60, 64: $BC < 25$ and does not meet requirement 3

68: $BC - 25 = 48 - 25 = 23$ does not divide $10B + C = 68$ and does not meet requirement 2

72: $BC < 25$ and does not meet requirement 3

76: $BC - 25 = 42 - 25 = 17$ does not divide $10B + C = 76$ and does not meet requirement 2

80: $BC < 25$ and does not meet requirement 3

84: $BC - 25 = 32 - 25 = 7$ divides $10B + C = 84$ and $25 \leq BC = 32 < 250$, meeting both requirements.

Our equation gives us $A = 3$ and a resulting number, $\boxed{N = 384}$.

4. This is just a simple unit conversion:

$$1 \text{ m}^3 \times \frac{100^3 \text{ cm}^3}{1 \text{ m}^3} \times \frac{1 \text{ L}}{1000 \text{ cm}^3} \times \frac{29 \text{ g}}{22 \text{ L}} = \boxed{1318 \text{ g}}$$

5. We first need to find the circumferences of the tires: 3.5π ft and 4.75π ft. Then we will do find how many revolutions each tire makes in 5000 miles by converting units like the previous problem:



Math Olympiad and Problem Solving Programs
E130 - Honors Geometry Problem Solving
Problem Set 12.2 - Arithmetic

Name:

Date:

Larger tire:

$$5000 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ revolution}}{4.75\pi \text{ ft}} = 177030$$

Smaller tire:

$$5000 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ revolution}}{3.5\pi \text{ ft}} = 2402184$$

The difference is $2402184 - 1770030 = \boxed{632,154}$.

6. $\boxed{1.3}$

7. $\boxed{85.32}$

8. Let A, B be the two different digits. We can either have 2 of each digit or 3 of one and 1 of the other. If we have 2 of each digit, then $2A + 2B = 9$, which is a contradiction since 9 is odd. We can conclude that we have 3 of one digit and 1 of the other. Without loss of generality, let's assume that the number has the digit A 3 times and the digit B once. Then $3A + B = 9$. Then the possibilities for pairs (A, B) are $(1, 6), (2, 3)$. If our number has digits 1, 6 then the number must be 1116 since it must be even. But this is twice 558 which does not satisfy these conditions so we know that $(A, B) = (2, 3)$. The possible numbers now are 3222, 2322, 2232. Simple checking gives us $\boxed{2232}$ as the solution.

9. $\boxed{51}$

10. Let M, T, S be Marla's, Tim's, and Scott's scores respectively. The information tells us $M = 2T$ and $M = \frac{4}{5}S$. The first thing we notice is that Tim's score is the lowest so we will find the lowest score he could have and then plug in to find Scott's score. The first equation tells us that Marla's score is even. The second equation tells us that Marla's score is actually a multiple of 4 ($5M = 4S$). In order for Marla's score to be a multiple of 4, from the first equation Tim's score must be even. The first even score greater than or equal to 17 is 18. Then Marla must score at least 36 and Scott must score at least $\boxed{45}$.