

1. (a) $27^{\frac{1}{2}} \times 3^{\frac{1}{2}} = (3^3)^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^{\frac{3}{2}} \times 3^{\frac{1}{2}} = 3^{(\frac{3}{2}+\frac{1}{2})} = 3^2 = \boxed{9}$

(b) $32^{\frac{3}{5}} \div 16^{\frac{1}{4}} = (2^5)^{\frac{3}{5}} \div (2^4)^{\frac{1}{4}} = 2^3 \div 2 = 2^{(3-1)} = 2^2 = \boxed{4}$

(c) $2^{-3} \times 16^{\frac{3}{4}} = 2^{-3} \times (2^4)^{\frac{3}{4}} = 2^{-3} \times 2^3 = 2^{(-3+3)} = 2^0 = \boxed{1}$

2. Simplify

(a) $\sqrt[3]{a} \div \sqrt[5]{a^2} \times (a^{-1})^{\frac{1}{2}} = a^{\frac{1}{3}} \div (a^2)^{\frac{1}{5}} \times a^{-\frac{1}{2}} = a^{\frac{1}{3}} \div a^{\frac{2}{5}} \times a^{-\frac{1}{2}} = a^{(\frac{1}{3}-\frac{2}{5}-\frac{1}{2})} = \boxed{a^{-\frac{17}{30}}}$

(b)

$$\begin{aligned} 12^{2x+3} \times 6^{x-5} \times 8^{-x-1} &= (2^2 \times 3)^{2x+3} \times (2 \times 3)^{x-5} \times (2^3)^{-x-1} \\ &= (2^2)^{2x+3} \times 3^{2x+3} \times 2^{x-5} \times 3^{x-5} \times 2^{-3x-3} \\ &= 2^{4x+6} \times 3^{2x+3} \times 2^{x-5} \times 3^{x-5} \times 2^{-3x-3} \\ &= 2^{4x+6+x-5-3x-3} \times 3^{2x+3+x-5} \\ &= \boxed{2^{2x-2} \times 3^{3x-2}} \end{aligned}$$

(c)

$$\begin{aligned} \frac{81^{x+1} - 9^{2x+1}}{3^{x-4} \times 27^{x+2}} &= \frac{(3^4)^{x+1} - (3^2)^{2x+1}}{3^{x-4} \times (3^3)^{x+2}} \\ &= \frac{3^{4x+4} - 3^{4x+2}}{3^{x-4} \times 3^{3x+6}} \\ &= \frac{3^{4x} \times 3^4 - 3^{4x} \times 3^2}{3^{x-4+3x+6}} \\ &= \frac{3^{4x} \times 3^2 \times (3^2 - 1)}{3^{4x+2}} \\ &= \frac{3^{4x+2} \times (3^2 - 1)}{3^{4x+2}} \\ &= \frac{3^{4x+2}}{3^{4x+2}} \times \frac{3^2 - 1}{1} \\ &= 3^2 - 1 \\ &= \boxed{8} \end{aligned}$$

3. (a) $7^{-1} \times 17^0 \times 49^{\frac{3}{2}} = 7^{-1} \times 1 \times (7^2)^{\frac{3}{2}} = 7^{-1} \times 7^3 = 7^{-1+3} = 7^2 = \boxed{49}$

(b) $8^{-\frac{2}{3}} \times 4^{\frac{1}{2}} \div 2^{-3} = (2^3)^{-\frac{2}{3}} \times (2^2)^{\frac{1}{2}} \div 2^{-3} = 2^{-2} \times 2^1 \div 2^{-3} = 2^{-2+1-(-3)} = 2^2 = \boxed{4}$

(c) $(\frac{1}{27})^{-\frac{4}{3}} \times 81^{\frac{1}{4}} \div 3^{-1} = (3^{-3})^{-\frac{4}{3}} \times (3^4)^{\frac{1}{4}} \div 3^{-1} = 3^4 \times 3^1 \div 3^{-1} = 3^{4+1-(-1)} = \boxed{3^6 = 729}$

(d) $8^{\frac{2}{3}} \times 4^{-1} \times 512^{-\frac{4}{3}} = (2^3)^{\frac{2}{3}} \times (2^2)^{-1} \times (2^9)^{-\frac{4}{3}} = 2^2 \times 2^{-2} \times 2^{-12} = 2^{2-2-12} = 2^{-12} = \boxed{\frac{1}{2^{12}} = \frac{1}{4096}}$

(e) $49^{\frac{1}{4}} \times 7^{\frac{1}{3}} \div 7^{-\frac{1}{6}} = (7^2)^{\frac{1}{4}} \times 7^{\frac{1}{3}} \div 7^{-\frac{1}{6}} = 7^{\frac{1}{2}} \times 7^{\frac{1}{3}} \div 7^{-\frac{1}{6}} = 7^{(\frac{1}{2}+\frac{1}{3}-(-\frac{1}{6}))} = \boxed{7}$

4. (a) $a^{\frac{1}{2}} \times a^{\frac{1}{3}} \times a^{\frac{1}{6}} = a^{(\frac{1}{2} + \frac{1}{3} + \frac{1}{6})} = \boxed{a}$
 (b) $a^3 \div a^{-4} \times a^2 = a^{3 - (-4) + 2} = \boxed{a^9}$
 (c) $12a^{-4} \div 4a^{-6} = 3a^{-4 - (-6)} = \boxed{3a^2}$
 (d) $16a^{-\frac{5}{2}} \div 4a^{-\frac{3}{2}} = 4a^{(-\frac{5}{2} - (-\frac{3}{2}))} = \boxed{4a^{-1}}$
 (e) $(a^{\frac{1}{3}} \times b^{\frac{2}{5}})^{15} = (a^{\frac{1}{3}})^{15} \times (b^{\frac{2}{5}})^{15} = \boxed{a^5 b^6}$
 (f) $(a^{-\frac{1}{4}} \div a^{\frac{3}{8}})^{24} = (a^{-\frac{1}{4}})^{24} \div (a^{\frac{3}{8}})^{24} = a^{-6} \div a^9 = a^{-6-9} = \boxed{a^{-15}}$
 (g) $\sqrt[4]{a^3} \times \sqrt{a} \div \sqrt[3]{a^2} = (a^3)^{\frac{1}{4}} \times a^{\frac{1}{2}} \div (a^2)^{\frac{1}{3}} = a^{\frac{3}{4}} \times a^{\frac{1}{2}} \div a^{\frac{2}{3}} = a^{(\frac{3}{4} + \frac{1}{2} - \frac{2}{3})} = \boxed{a^{\frac{7}{12}}}$
 (h) $\sqrt[6]{a^{x+2}} \div \sqrt[9]{a^{x+3}} = (a^{x+2})^{\frac{1}{6}} \div (a^{x+3})^{\frac{1}{9}} = a^{(\frac{x}{6} + \frac{1}{3})} \div a^{(\frac{x}{9} + \frac{1}{3})} = a^{(\frac{x}{6} + \frac{1}{3} - \frac{x}{9} - \frac{1}{3})} = \boxed{a^{\frac{x}{18}}}$

5. (a)

$$\begin{aligned} 3^{x+4} \times 5^{x+1} \times 15^{2x-1} &= 3^{x+4} \times 5^{x+1} \times (3 \times 5)^{2x-1} \\ &= 3^{x+4} \times 5^{x+1} \times 3^{2x-1} \times 5^{2x-1} \\ &= 3^{x+4+2x-1} \times 5^{x+1+2x-1} \\ &= \boxed{3^{3x+3} \cdot 5^{3x}} \end{aligned}$$

(b)

$$\begin{aligned} 5^{x+7} \times 25^{2x-1} \div 125^{2-x} &= 5^{x+7} \times (5^2)^{2x-1} \div (5^3)^{2-x} \\ &= 5^{x+7} \times 5^{4x-2} \div 5^{6-3x} \\ &= 5^{x+7+4x-2-6+3x} \\ &= \boxed{5^{8x-1}} \end{aligned}$$

(c)

$$\begin{aligned} 6^{3x+1} \times 8^{x-1} \times 24^{3x-1} &= (2 \times 3)^{3x+1} \times (2^3)^{x-1} \times (2^3 \times 3)^{3x-1} \\ &= 2^{3x+1} \times 3^{3x+1} \times 2^{3x-3} \times (2^3)^{3x-1} \times 3^{3x-1} \\ &= 2^{3x+1} \times 3^{3x+1} \times 2^{3x-3} \times 2^{9x-3} \times 3^{3x-1} \\ &= 2^{3x+1+3x-3+9x-3} \times 3^{3x+1+3x-1} \\ &= \boxed{2^{15x-5} \cdot 3^{6x}} \end{aligned}$$

(d)

$$\begin{aligned} 2^{x-1} \times 4^{3x-2} \div 32^{2x+1} &= 2^{x-1} \times (2^2)^{3x-2} \div (2^5)^{2x+1} \\ &= 2^{x-1} \times 2^{6x-4} \div 2^{10x+5} \\ &= 2^{x-1+6x-4-10x-5} \\ &= \boxed{2^{-3x-10}} \end{aligned}$$



Math Olympiad and Problem Solving Programs
E120 - Honors Algebra Problem Solving
Problem Set 29.2 - Simplifying Exponents

Name:

Date:

(e)

$$\begin{aligned}6^x \times 12^{2x+2} \div 27^x \times 32^{3x} &= (2 \times 3)^x \times (2^2 \times 3)^{2x+2} \div (3^3)^x \times (2^5)^{3x} \\&= 2^x \times 3^x \times (2^2)^{2x+2} \times 3^{2x+2} \div 3^{3x} \times 2^{15x} \\&= 2^x \times 3^x \times 2^{4x+4} \times 3^{2x+2} \div 3^{3x} \times 2^{15x} \\&= 2^{x+4x+4+15x} \times 3^{x+2x+2-3x} \\&= \boxed{2^{20x+4} \times 3^2}\end{aligned}$$

(f)

$$\begin{aligned}20^{x+3} \times 15^{2x+5} \div 6^{2x-1} &= (2^2 \times 5)^{x+3} \times (3 \times 5)^{2x+5} \div (2 \times 3)^{2x-1} \\&= (2^2)^{x+3} \times 5^{x+3} \times 3^{2x+5} \times 5^{2x+5} \div 2^{2x-1} \div 3^{2x-1} \\&= 2^{2x+6} \times 5^{x+3} \times 3^{2x+5} \times 5^{2x+5} \div 2^{2x-1} \div 3^{2x-1} \\&= 2^{2x+6-2x-1} \times 3^{2x+5-2x-1} \times 5^{x+3+2x+5} \\&= \boxed{2^7 \cdot 3^6 \cdot 5^{3x+8}}\end{aligned}$$