READ FIRST The sum of a finite geometric series is $s_n = \frac{a_1(1-r^n)}{1-r}$, where a_1 is the first term, r is the common ratio, and n is the number of terms in the series.

The sum of an infinite geometric series, which only exists if $|r| < 1$, is $s_n = \frac{a_1}{1-r}$.

1. (a) $\boxed{\frac{6141}{1024} \approx 6}$

   (b) This problem is tricky because it is not a geometric series as is. However, it is equivalent to $7n + (1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16})$. Now the second part of the sum is a geometric series and we can solve for it.

   We know that $a_1 = 1, n = 5$. Now $r = \frac{3}{2} \div 1 = \frac{3}{2}$. This gives us the sum to be $s_n = 7(5) + \frac{1(1-(\frac{3}{2})^5)}{1-\frac{3}{2}} = \boxed{\frac{365}{16} = 22.8125}$.

2. Find the sum of the infinite geometric series

   (a) $a_1 = 1, r = -\frac{1}{2}$ so we have $s_n = \frac{1}{1--\frac{1}{2}} = \boxed{\frac{2}{3} \approx 0.67}$.

   (b) $a_1 = -\frac{100}{9}, r = -\frac{3}{10}$ so we have $s_n = \frac{-\frac{100}{9}}{1--\frac{3}{10}} = \boxed{-\frac{1000}{117} \approx 8.54}$.

   (c) $a_1 = \frac{1}{\sqrt{2}}, r = \frac{1}{\sqrt{2}}$ so we have $s_n = \frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} = \boxed{\sqrt{2}+1 \approx 2.41}$.

   (d) $a_1 = \frac{2}{5}, r = \frac{2}{5}$ so we have $s_n = \frac{\frac{2}{5}}{1-\frac{2}{5}} = \boxed{\frac{2}{3} \approx 0.67}$.

3. (a) $0.\overline{36} = 0.36 + 0.0036 + 0.000036 + \cdots$ so $a_1 = 0.36, r = 0.01$ so we have $0.\overline{36} = \frac{0.36}{1-0.01} = \boxed{\frac{4}{11}}$

   (b) $1.3\overline{8} = 1.3 + (0.08 + 0.008 + 0.0008 + \cdots)$, 1.3 plus a series with $a_1 = 0.08, r = 0.1$ so $1.3\overline{8} = 1.3 + \frac{0.08}{1-0.08} = \boxed{\frac{25}{18}}$.

4. If each year the population increases by 1.3%, the population n years from now is represented by the geometric sequence:

   $$250000, 250000(1.013), 250000(1.013)^2, \ldots, 250000(1.013)^n$$

   This means the population 30 years from now will be $250000(1.013)^{30} = \boxed{368{,}318}$.

5. Since the interest is compounded monthly, every month the money increases by 6%. This is represented by the geometric sequence:

   $$1000, 1000(1.06), 1000(1.06)^2, \ldots, 1000(1.06)^n, \ldots$$

   This means that in n months, there principal will increase to $1000(1.06)^n$. In 10 years we will have compounded $12 \times 10 = 120$ months' worth of interest, which is $1000(1.06)^{120} = \boxed{\$1{,}088{,}187.74}$.

6. Since the interest is compounded quarterly, the total money n quarters from now is represented by the geometric sequence:

$$2500, 2500(1.08), 2500(1.08)^2, \ldots, 2500(1.08)^n$$

20 years equates to $4 \times 20 = 80$ quarters, so our principal will have increased to $2500(1.08)^{80} =$ $\boxed{\$1{,}179{,}887.09}$.

7. A is the sum of a finite geometric series so we use that formula. $a_1 = 100(1 + \frac{0.06}{12}), r = (1 + \frac{0.06}{12}), n = 60$. This gives us our sum to be:

$$\begin{aligned} \frac{a_1(1 - r^n)}{1 - r} &= \frac{100(1 + \frac{0.06}{12})[1 - (1 + \frac{0.06}{12})^{60}]}{1 - (1 + \frac{0.06}{12})} \\ &= \frac{100(1.005)(1 - 1.005^{60})}{-0.005} \\ &\approx \boxed{\$7,011.89} \end{aligned}$$

The last calculation must be done with a calculator, obviously.

8. The total income is a geometric series, $0.01 + 0.02 + 0.04 + \cdots + 0.01(2)^{29}$. Notice the power goes up to 29, not 30, because the first day is the starting wage which is not doubled. This is the sum of a finite geometric series. $a_1 = 0.01, r = 2, n = 30$ (there are still 30 terms).

$s_{30} = \dfrac{0.01(1 - 2^{30})}{1 - 2} = \boxed{\$10{,}737{,}418.23}$.

9. The total compensation is the sum of the salaries from each year, represented by the following geometric series:

$$\begin{aligned} s_n &= 30000 + 30000(1.05) + 30000(1.05)^2 + \cdots + 30000(1.05)^{39} \\ &= \frac{30000(1 - 1.05^{40})}{1 - 1.05} \\ &= \boxed{\$3,623,993,23} \end{aligned}$$

10. The ball first travels down 16 feet, and then it bounces back up 16(0.81) feet and back down 16(0.81) feet, and then it bounces back up $16(0.81)^2$ feet and back down $16(0.81)^2$ feet, and so on.

This gives us a geometric sequence $16, 12.96, 10.4976, \ldots, 16(0.81)^{n-1}, \ldots$. The total distance traveled looks like the sum of the series, but since for every rebound the ball come up and goes down the same distance, it is really twice the series with the exception of the first drop.

Thus the ball really travels $2(16 + 12.96 + 10.4976 + \cdots + 16(0.81)^{n-1} + \cdots) - 16 = 2(\frac{16}{1-0.81}) - 16 = \boxed{\frac{2896}{19} \approx 152.42 \text{ ft}}$.