



Math Olympiad and Problem Solving Programs  
E120 - Honors Algebra Problem Solving  
Problem Set 27.1 - Arithmetic Series

Name:

Date:

1. (a)  $\boxed{819}$

(b) The given information is  $a_1 = 7, d = -3, n = 14$ . We need to find  $a_{14}$ . We first find  $a_n$ :

$$\begin{aligned}a_n &= a_1 + d(n - 1) \\ &= 7 - 3(n - 1) \\ &= 7 - 3n + 3 \\ &= 10 - 3n\end{aligned}$$

Plug in  $n = 14$ :

$$a_{14} = 10 - 3n = 10 - 3(14) = 10 - 42 = -32$$

Then the sum is  $(a_1 + a_n) \cdot \frac{n}{2}$ :

$$s_{14} = (7 - 32) \cdot \frac{14}{2} = -25 \cdot 7 = \boxed{-175}$$

(c) The given information is  $a_1 = \frac{1}{2}, d = \frac{1}{3}, a_n = \frac{19}{2}$ . We need to find  $n$ . We first find the general solution,  $a_n$ :

$$\begin{aligned}a_n &= a_1 + d(n - 1) \\ &= \frac{1}{2} + \frac{1}{3}(n - 1) \\ &= \frac{1}{2} + \frac{n}{3} - \frac{1}{3} \\ &= \frac{n}{3} + \frac{1}{6}\end{aligned}$$

Plug in  $a_n = \frac{19}{2}$ :

$$\begin{aligned}\frac{19}{2} &= \frac{n}{3} + \frac{1}{6} \\ 57 &= 2n + 1 \\ 56 &= 2n \\ n &= 28\end{aligned}$$

Then the sum is  $(a_1 + a_n) \cdot \frac{n}{2}$ :

$$s_{28} = \left(\frac{1}{2} + \frac{19}{2}\right) \cdot \frac{28}{2} = 10 \cdot 14 = \boxed{140}$$



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2. The first 50 odd numbers are  $1, 3, 5, \dots$ . This gives us the following information:  $a_1 = 1, d = 2, n = 50$ . We now need to find  $a_{50}$ . We first find  $a_n$ :

$$\begin{aligned}a_n &= a_1 + d(n - 1) \\ &= 1 + 2(n - 1) \\ &= 1 + 2n - 2 \\ &= 2n - 1\end{aligned}$$

Plug in  $n = 50$ :

$$a_{50} = 2(50) - 1 = 99$$

Then the sum is  $(a_1 + a_n) \cdot \frac{n}{2}$ :

$$s_{50} = (1 + 99) \cdot \frac{50}{2} = 100 \cdot 25 = \boxed{2500}$$

3.  $\boxed{-50}$

4. The information given is  $a_1 = 7, n = 15, s_{15} = -210$ . We use the formula for the sum of the series:

$$\begin{aligned}s_n &= (a_1 + a_{15}) \cdot \frac{n}{2} \\ -210 &= (7 + a_{15}) \cdot \frac{15}{2} \\ -28 &= 7 + a_{15} \\ a_{15} &= -35\end{aligned}$$

Now we can use the general term  $a_n$  to find the common difference:

$$\begin{aligned}a_n &= a_1 + d(n - 1) \\ -35 &= 7 + d(15 - 1) \\ -35 &= 7 + 14d \\ -42 &= 14d \\ d &= \boxed{-3}\end{aligned}$$

5. The information given is  $s_5 = 70$ ,  $s_{10} = 210$ . If we plug this information into the formula for sum of an arithmetic series, we get the following system of equations:

$$\begin{cases} 70 &= (a_1 + a_5) \cdot \frac{5}{2} \\ 210 &= (a_1 + a_{10}) \cdot \frac{10}{2} \end{cases}$$

In the form shown it cannot be solved, but it can be shown that  $a_5 = a_1 + 4d$  and  $a_{10} = a_1 + 9d$ . We can plug this into our system and we now have a system with only 2 unknowns ( $a_1, d$ ):

$$\begin{cases} 70 &= (a_1 + a_1 + 4d) \cdot \frac{5}{2} \\ 210 &= (a_1 + a_1 + 9d) \cdot \frac{10}{2} \end{cases}$$

Solving gives us  $a_1 = \frac{42}{5} = 8.4$ .

6. The information given is  $n = 20$ ,  $d = 3$ ,  $s_n = 650$ . We use the sum of an arithmetic sequence:

$$650 = (a_1 + a_n) \cdot \frac{20}{2}$$

$$650 = (a_1 + a_n) \cdot 10$$

$$65 = a_1 + a_n$$

But we know  $a_n = a_1 + d(n - 1) = a_1 + 3(n - 1) = a_1 + 57$  so we can plug this in:

$$65 = a_1 + a_1 + 57$$

$$65 = 2a_1 + 57$$

$$8 = 2a_1$$

$$a_1 = \boxed{4}$$

7.  $\boxed{1220}$

8. This is an arithmetic series with  $n = 30$ ,  $a_1 = 20$ ,  $d = 4$ . We need to find  $a_{30}$ :

$$a_n = a_1 + d(n - 1)$$

$$= 20 + 4(n - 1)$$

$$= 20 + 4n - 4$$

$$= 16 + 4n$$

Plug in  $n = 30$ :

$$a_{30} = 16 + 4(30) = 16 + 120 = 136$$

Now we can find the sum:

$$s_{30} = (20 + 136) \cdot \frac{30}{2} = (156) \cdot 15 = \boxed{2340}$$



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9. 405

10. This is an arithmetic series with  $n = 17$ ,  $a_1 = 4.9$ ,  $d = 9.8$  and we are looking for  $s_{17}$ . We need to find  $a_{17}$ :

$$\begin{aligned} a_n &= a_1 + d(n - 1) \\ &= 4.9 + 9.8(n - 1) \\ &= 4.9 + 9.8n - 9.8 \\ &= 9.8n - 4.9 \end{aligned}$$

Plug in  $n = 17$ :

$$a_{17} = 9.8(17) - 4.9 = 161.7$$

Now we can find the sum:

$$s_{17} = (4.9 + 161.7) \cdot \frac{17}{2} = 166.6 \cdot \frac{17}{2} = 83.3 \cdot 17 = \boxed{1416.1 \text{ m}}$$