1. (a) $\boxed{x < 9}$
   (b) $\boxed{x \leq 9}$
   (c) $\boxed{x \geq -19}$
   (d) $\boxed{x > 0}$

2. (a) $\boxed{x < \frac{17}{4}}$
   (b) $\boxed{x \geq -\frac{1}{3}}$
   (c) $\boxed{x > \frac{19}{12}}$
   (d) $\boxed{x < -\frac{15}{22}}$

3. (a) There are two ways of solving this:

   First Method:

$$4 - \frac{1}{4} \leq 5x$$
$$\frac{15}{4} \leq 5x$$
$$\boxed{\frac{3}{4} \leq x \text{ -OR- } x \geq \frac{3}{4}}$$

   Second Method:

$$-5x \leq \frac{1}{4} - 4$$
$$-5x \leq -\frac{15}{4}$$
$$\boxed{x \geq \frac{3}{4}}$$

   Notice that, in the Second Method, when we divided by -5 we switched the inequality from $\leq$ to $\geq$. This is because any time you multiply or divide by a negative number, you must switch the inequality.

   (b) $\boxed{x \leq -\frac{17}{9}}$
   (c) $\boxed{x \leq -\frac{18}{5}}$
   (d) $\boxed{x > -\frac{3}{4}}$

4. (a) $\boxed{x \geq 16}$

(b) Again there are two ways of solving this inequality:

First Method:

$$\begin{aligned} -x + \frac{5}{2} &\geq \frac{x}{3} + 1 \\ \frac{5}{2} - 1 &\geq \frac{x}{3} + x \\ \frac{3}{2} &\geq \frac{4x}{3} \\ \frac{3}{2} \cdot \frac{3}{4} &\geq x \\ \boxed{\frac{9}{8} \geq x \text{ -OR- } x \leq \frac{9}{8}} \end{aligned}$$

Second Method:

$$\begin{aligned} 6\left[-\frac{1}{2}(2x-5)\right] &\geq 6\left[\frac{1}{3}(x+3)\right] \\ -3(2x-5) &\geq 2(x+3) \\ -6x + 15 &\geq 2x + 6 \\ 9 &\geq 8x \\ \boxed{\frac{9}{8} \geq x \text{ -OR- } x \leq \frac{9}{8}} \end{aligned}$$

If you dislike fractions, just multiply both sides by the common denominator.

(c) This problem may have been graded incorrectly.

$$\begin{aligned} x - \frac{3}{4} &< \frac{7x}{2} + 21 \\ 4\left(x - \frac{3}{4}\right) &< 4\left(\frac{7x}{2} + 21\right) \\ 4x - 3 &< 14x + 84 \\ -10x &< 87 \\ \boxed{x < -\frac{87}{10}} \end{aligned}$$

(d)

$$\begin{aligned} 6\left[\frac{3}{2}(4x-7)\right] &\le 6\left[\frac{2}{3}(2x+5)\right] \\ 9(4x-7) &\le 4(2x+5) \\ 36x-63 &\le 8x+20 \\ 28x &\le 83 \\ \boxed{x \le \frac{83}{28}} \end{aligned}$$

5. We first solve the inequality for x:

$$\begin{aligned} -3x &\le -22 \\ x &\ge \frac{22}{3} = 7\frac{1}{3} \end{aligned}$$

The least integer value of x would be $\boxed{8}$.

6. $\boxed{73}$

7. $\boxed{(x, y) = (7, 56), (14, 49), (21, 42), (28, 35)}$

8. $\boxed{(x, y) = (30, 3), (27, 9), (24, 15), (21, 21)}$

9. (a) To find the greatest value of $y^2 - x^2$, we want the largest value for y^2 and the smallest value for x^2. This happens when $(x, y) = (1, 7)$; we get $7^2 - 1^2 = 49 - 1 = \boxed{48}$.

(b) To find the smallest value of $y^2 - x^2$, we want the smallest value for y^2 and the largest value for x^2. This happens when $(x, y) = (3, 2)$; we get $2^2 - 3^2 = 4 - 9 = \boxed{-5}$

10. (a) To find the greatest value of $xy - \frac{1}{y}$, xy must be positive since $|xy| > \left|\frac{1}{y}\right|$. Since $y < 0$, x must be < 0 in order for xy to be positive. $\frac{1}{y}$ is so miniscule that we want the largest $|y|$. This happens when $(x, y) = (-5, -8)$; we get $(-5)(-8) - \frac{1}{-8} = 40 + \frac{1}{8} = \boxed{\frac{321}{8} = 40\frac{1}{8}}$.

(b) To find the smallest value of $xy - \frac{1}{y}$, again $\frac{1}{y}$ is miniscule enough that we want the largest negative value for xy. This happens when $(x, y) = (9, -8)$; we get $(9)(-8) - \frac{1}{-8} = -72 + \frac{1}{8} =$ $\boxed{-\frac{575}{8} = -71\frac{7}{8}}$